Conformal boundaries in pseudo-Riemannian geometry III

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If $\Gamma \subset \mathrm{PO}(p+1, q+1)$ is discrete, and $M = \Gamma \backslash \Omega$ is Kleinian, then $\partial_c \hat{M}$ is homeomorphic to $\Gamma \backslash \partial\hat{\Omega}$, and the P-action is the right action of P on $\Gamma \backslash \partial\hat{\Omega}$.

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- *Let* (M, g) *be a complete flat manifold which is not the Euclidean space. Then* $\partial_c \hat{M}$ *is identified to* $\Gamma \backslash P$*, for some nontrivial* Γ*. In particular the* P *action is nowhere free.*

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- *Let* (M, g) *be a complete hyperbolic manifold of finite volume. Then* $\partial_c \hat{M}$ *is a smooth manifold of dimension* $n - 1 + \dim P$*, on which the* P*-action is minimal (all orbits are dense).*

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- Hence there is a 1-Lipschitz P-equivariant extension $\hat{\sigma}_{ext} : \hat{M}_c \to \hat{N}_c$, which restricts to a P-equivariant boundary map

$$\partial \hat{\sigma} : \partial_c \hat{M} \to \hat{N}_c.$$

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- That's why we introduce the regular set $\hat{\Lambda}_{reg} = \partial\hat{\sigma}^{-1}(\hat{N})$.
- Observe that the regular set depends on the embedding σ we are considering.

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3. *The group P acts freely and properly on $\hat{\Lambda}_{reg} \cup \hat{M}$.*

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- It follows clearly from the previous proposition that if for some conformal structure $(M, [g])$, the action of the group P is free and proper on no nonempty open subset of $\partial_c \hat{M}$, then $(M, [g])$ is conformally maximal.

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2. *All complete manifolds of constant curvature -1 and finite volume (again of dimension $n \geq 3$).*

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- This theorem is true with the same proof for any Cartan geometry.

Just for fun....

- We just saw that if τ is a nontrivial translation of $\mathbb{R}^n$, and Γ is the subgroup generated by Γ, then $\mathbf{E}^n/\Gamma$ is conformally maximal ($n \geq 3$).
- But... $\mathbf{E}^{1,n-1}/\Gamma$ is not conformally maximal.
- On the other hand $\mathbf{E}^n/\Gamma$ (or equivalently $\mathbf{E}^{1,n-1}/\Gamma$) is projectively maximal.

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In particular, if N is compact, (N, h) is conformally diffeomorphic to $\mathbf{Ein}^{p,q}$.

- Application to $\mathbf{S}^k \times \mathbf{H}^{\mathbf{n}-\mathbf{k}}$, $n - 1 \geq k \geq 1$, and $\mathbf{AdS}^{\mathbf{k}} \times \mathbf{S}^{\mathbf{n}-\mathbf{k}}$, $n - 1 \geq k \geq 2$.

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- Check that for every open set U, $\pi(U \setminus \{\nu\}) = \mathbf{T}^n$, so that π does not extend even continuously to $\mathbf{S}^n$.
- In the previous theorem, it was very important to deal with conformal embeddings, and not merely with immersions (find where we used it).

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Let (L, g) and (N, h) be two connected, compact, n-dimensional Riemannian manifolds, $n \geq 3$. Let $\Lambda \subset L$ be a closed subset such that $\mathcal{H}^{n-1}(\Lambda) = 0$. Let $\sigma : L \setminus \Lambda \to N$ a conformal embedding. Then σ extends to a conformal diffeomorphism

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- This theorem looks like the previous one, but is harder to prove, even when $\lambda = \{x_0\}$, because the manifold $L \setminus \Lambda$ is no longer an open subset of the model. Actually, the Cauchy boundary is here useless to solve the problem.

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- These methods don't use really the specificity of conformal geometry, and are actually easy to generalize to other "Cartan geometries" (*CR*-structures, projective structures etc...)
- The main drawback of the Cauchy boundary is that we can determine it only for few classes of spaces (here, mostly conformally flat ones).
- To get results of the same flavor for general conformal structures, the use of the normal Cartan connection is still powerfull, but proofs are more involved, and require other tools (use of conformal geodesics etc...).