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Jan Gregorovič	Martin Schnabl
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Josef Janyška	Josef Šilhan
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Tom Lada	Jan Vysoký
Helder Larraguivel	Alice Waterhouse
Roman Lávička	Travis Willse
Hong Van Le	Henrik Winther
Jerzy Lewandowski	Artur Woike
Antonella Marchesiello	Lenka Zalabová
Martin Markl	Petr Zima
Michal Marvan	Alexander Zuevsky
Alexandr Medvedev	Vojtěch Žádník

ANNOUNCED LECTURES

A. INVITED LECTURES

Maciej Dunajski: *Einstein–Weyl geometry, dispersionless integrable systems and twistor theory*

Evgeny Ferapontov: *Dispersionless integrable systems: classification, differential geometry, deformations*

Collin Guillarmou: *Mathematical aspects of 2D conformal field theory and renormalisations*

Marc Herzlich: *Some old and new issues about mass in mathematical relativity*

Albrecht Klemm: *Topological String theory and String dualities*

Jerzy Lewandowski: *Non-expanding horizons including near horizon geometries as related subject*

B. OTHER LECTURES

Lashi Bandara: *Spectral flows and the Riesz continuity of the Dirac operator with respect to boundary value perturbations*

Klaus Bering: *Batalin–Fradkin–Vilkovisky (BFV) quantization of bosonic string theory*

Andrzej Borowiec: *Palatini cosmology and different frames*

Andreas Čap: *PCS-structures and differential complexes*

Goce Chadzitaskos: *On the principle of least action*

Boris Doubrov: *Degenerate pseudo-product and CR structures*

Zdeněk Dušek: *Homogeneous geodesics in homogeneous Finsler spaces*

Fotis Farakos: *N Goldstini*

Jan Gregorovič: *Non-involutive symmetries on parabolic contact geometries*

Matthias Hammerl: *Conformal Patterson–Walker metrics and Fefferman–Graham ambient spaces*

Ondřej Hulík: *Multi-centered AdS3 solutions from Virasoro conformal blocks*

Martin Kolář: *Nonlinear symmetries and gap phenomena for polynomial models in \mathbb{C}^3 .*

Ilya Kossovskiy: *On the CR-embeddability of real hypersurfaces into quadrics and CR-complexity*

Pavel Kočí: *Supersymmetry breaking and higher derivative terms*

Boris Kruglikov: *Differential invariants: calculus and count*

Svatopluk Krýsl: *Symplectic dirac operator for KK-theory*

Tom Lada: *TBA*

Roman Lávička: *The skew Pieri rule for the symplectic group*

Hong Van Le: *Cramér–Rao inequality and machine learning*

Antonella Marchesiello: *Superintegrable 3D systems in a magnetic field and Cartesian separation of variables*

Michal Marvan: *Spacetimes with symmetry. From scalar invariants to exact solutions.*

Alexandr Medvedev: *Sub-Riemannian Engel structures: abnormal geodesics and integrability*

Giovanni Moreno: *Second-order PDEs with large groups of symmetries*

Jan Novák: *K-essence models and late time cosmic acceleration*

Pavle Pandžić: *Dirac cohomology of unitary highest weight modules*

Prim Plansangkate: *Skyrmions, multi-instantons and $SU(\infty)$ -Toda equation*

Tomáš Procházka: *W symmetry and affine Yangian*

Ján Pulmann: *Minimal model of loop homotopy Lie algebras*

Tomáš Salač: *Elliptic complex on the Grassmanian of oriented 2-planes in \mathbb{R}^{n+2}*

Martin Schnabl: *TBA (Some topic in String Field Theory)*

Petr Somberg: *TBA*

Vladimír Souček: *Applications of the F-method in projective and Grassmannian geometry*

Marek Szydłowski: *Geometry of dynamics – cosmological models as dynamical systems of Newtonian type*

Josef Šilhan: *On projective metrizable in low dimensions*

Michal Širaň: *Integration of Courant algebroids*

Libor Šnobl: *Third-order superintegrable systems with potentials satisfying only nonlinear equations*

Dennis The: *On C-class equations*

Jiří Tolar: *From Lie gradings to gradings of operator algebras*

Aleksy Tralle: *On homology Smale-Barden manifolds with K-contact and Sasakian structures*

Alena Vanžurová: *(poster) Reconstruction of an Affine Connection in Generalized Fermi Coordinates*

Travis Willse: *Almost Einstein (2, 3, 5) conformal structures*

Henrik Winther: *Jacobi Identity via Lie Algebra Cohomology*

Artur Woike: *New constructions of symplectically fat bundles*

Petr Zima: *Integrability conditions for Killing equations on homogeneous spaces*

Alexander Zuevsky: *Higher genus partition and correlation functions for vertex superalgebras and modular form identities*

Vojtěch Žádník: *Tractor Frenet formulas in conformal and projective geometry*

Special Sessions

Within the programme, there will be two special sessions devoted to the memory of our distinguished young colleague, Martin Doubek, who passed away in a tragic accident last summer. Watch the programme board for further details.

C. ABSTRACTS

Lashi Bandara: *Spectral flows and the Riesz continuity of the Dirac operator with respect to boundary value perturbations*

We study the Atiyah–Singer Dirac operator on smooth Riemannian Spin manifolds with smooth compact boundary. Under lower bounds on injectivity radius and bounds on the Ricci curvature and its first derivatives, we demonstrate that this operator is stable in the Riesz topology under bounded perturbations of local boundary conditions. Our work is motivated by the spectral flow and its connection to the Riesz topology. These results are obtained by obtaining similar results for a more wider class of elliptic first-order differential operators on vector bundles satisfying certain general curvature conditions.

Klaus Bering: *Batalin–Fradkin–Vilkovisky (BFV) quantization of bosonic string theory*

In standard string theory textbooks, such as e.g. Green–Schwarz–Witten, Polchinski, or Blumenhagen–Lust–Theisen, the Faddeev–Popov (FP) ghost action is treated covariantly, but the relative normalization of the action between the matter and the ghost sector is fudged/not explained. This requires Becchi–Rouet–Stora–Tyutin (BRST) formulation, but the BRST formulation in these textbooks is only developed in conformal gauge. (The indeterminacy of the action normalization is indirectly related to the indeterminacy of the normalization the path integral measure.) This talk makes clear that the BRST formulation works for arbitrary world-sheet (WS) metric. In particular, we recover a famous factor $1/2$ in the BRST charge.

Andreas Čap: *PCS-structures and differential complexes*

This talk reports on joint work with T. Salac (Prague). PACS-structures are first order G -structures determined by contact gradings of a simple Lie algebra. Each such structure comes with an underlying almost conformally symplectic structure, if this underlying structure is conformally symplectic, then we talk about a PCS-structure. I'll briefly recall the basic features of these structures and their relation to special symplectic connections and to parabolic contact structures. In the second part of the talk, I will discuss how invariant differential complexes on parabolic contact structures can be used to construct complexes of natural differential operators on manifolds endowed with a special symplectic connection and, in some cases, with a more general instance of a PCS-structure.

Goce Chadzitaskos: *On the principle of least action*

We present the principle of least action from slightly different point of view. Moreover we present a possible connection of the principle of least action with de Broglie wave equation.

Boris Doubrov: *Degenerate pseudo-product and CR structures*

We discuss different notions of degeneracy for pseudo-product and CR structures. We show that under certain weaker notions of non-degeneracy we can still construct some algebraic structures resembling and generalizing the Tanaka symbol of non-degenerate vector distributions.

Zdeněk Dušek: *Homogeneous geodesics in homogeneous Finsler spaces*

A geodesic is homogeneous if it is an orbit of a 1-parameter group of isometries or of affine transformations. The existence of a homogeneous geodesic in any homogeneous Riemannian manifold was proved by O. Kowalski and J. Szenthe (in 2000) using the algebraic methods on the Lie algebra \mathfrak{g} of the isometry group. In this case, the algebra \mathfrak{g} always admits a reductive decomposition.

In pseudo-Riemannian homogeneous manifolds (possibly nonreductive), similar existence result was obtained in the framework of affine geometry by the author (in 2009) using a new, more fundamental, affine method.

Recently, Z. Yan obtained similar existence result for homogeneous Finsler manifolds of odd

dimension by the modification of the original algebraic method and without a reference to the affine method and result. Also the positive existence result for Randers metrics was announced. In the present talk, these methods (algebraic and affine) will be illustrated and compared. It will be shown that the affine method can be used in the straightforward way to prove the mentioned Finslerian result (for odd dimension) more easily. Further, we prove the similar existence result also for homogeneous Finslerian manifolds of even dimension if the Finslerian metric is Berwald or reversible.

Fotis Farakos: *N Goldstini*

We study field theories with N extended nonlinearly realized supersymmetries, describing the couplings of models that contain N Goldstini. We discuss the equivalence of all these extended supersymmetry breaking models containing N Goldstini and reformulate the theory with N supersymmetries in terms of standard $N=1$ constrained superfields.

Evgeny Ferapontov: *Dispersionless integrable systems: classification, differential geometry, deformations*

Lecture 1: The method of hydrodynamic reductions In this lecture I will describe a general approach to integrability of multidimensional dispersionless PDEs based on the method of hydrodynamic reductions. This technique relies on the requirement that a given PDE possesses 'sufficiently many' multi-phase solutions obtained by decoupling the multidimensional PDE into a collection of commuting 1+1 dimensional systems of hydrodynamic type. The method will be illustrated by several examples including the dispersionless Kadomtsev-Petviashvili (dKP) equation.

Lecture 2: Classification results, differential-geometric aspects In this lecture I will apply the method of hydrodynamic reductions to describe integrable PDEs within various particularly interesting classes. The classification results link dispersionless integrability to such classical differential-geometric objects as submanifolds in Grassmannians, conformal structures in projective spaces, Einstein-Weyl geometry, self-dual geometry, etc.

Lecture 3: Dispersive deformations of dispersionless integrable systems In this final lecture I will describe a general 'quantisation' procedure which, starting with a dispersionless integrable system, reconstructs, in an integrable way, 'dispersive corrections'. This technique is based on the requirement that all hydrodynamic reductions of the dispersionless system are 'inherited' by its dispersive counterpart. The procedure will be illustrated by the classification of discrete Hirota-type equations in 3D.

Jan Gregorovič: *Non-involutive symmetries on parabolic contact geometries*

We discuss automorphisms ϕ of parabolic contact geometries with fixed point and tangent action $-id$ on the contact distribution at the fixed point, which we call symmetries on parabolic contact geometries. We show that if there is non-involutive symmetry ϕ , i.e., $\phi^2 \neq id$, then the parabolic contact geometry is flat in some neighborhood of the fixed point. This implies that the symmetric parabolic contact geometries, i.e., parabolic contact geometries on which each point is a fixed point of some symmetry, are either flat or one dimensional fiber bundles over symmetric spaces with invariant conformally symplectic structure.

Collin Guillarmou: *Mathematical aspects of 2D conformal field theory and renormalisations*

For Riemann surfaces, we will explain the relations between several objects which arise from renormalisations procedures in 2d Conformal Field Theory: the renormalised volume and the Liouville Quantum Field Theory partition function. The first one is from purely geometric nature while the second one uses probability. We will explain some relations with Teichmüller theory.

Matthias Hammerl: *Conformal Patterson-Walker metrics and Fefferman-Graham ambient spaces*

Situations in which Fefferman-Graham ambient metrics are known to exist as smooth Ricci flat

metrics and not just as an infinite order expansion are rare, and situations in which they can be described explicitly are even rarer. While global existence and an explicit formula has been known for conformally Einstein structures for a long time, only recently have other classes of conformal structures been shown to allow an explicit global ambient metric. The topic of the present talk are conformal Patterson–Walker metrics. Patterson–Walker metrics are generically not conformally Einstein, but the ambient metric exists to all orders and can be realized in a natural way. As an application of the concrete ambient metric realization it is shown that Patterson–Walker metrics have vanishing Q-curvature. This talk is based on joint work with Katja Sagerschnig (Polytecnico di Torino), Josef Šilhan (Masaryk University), Arman Taghavi-Chabert (University of Turin) and Vojtěch Žádník (Masaryk University).

Marc Herzlich: *Some old and new issues about mass in mathematical relativity*

The lectures will be focused on the mass of asymptotically flat Riemannian manifolds and subsequent generalizations. This is a now classical subject whose mathematical study began at the end of the 70's. However some important issues have remained open since, and a surprisingly rich link has been discovered recently with isoperimetric problems. The lectures will seek to introduce the subject and its main results. They will be organized in a 'survey' style, with not many proofs, but with (at least) a lot of open questions.

Ondřej Hulík: *Multi-centered AdS3 solutions from Virasoro conformal blocks*

We revisit the construction of multi-centered solutions in three-dimensional anti-de Sitter gravity in the light of the recently discovered connection between particle worldlines and classical Virasoro conformal blocks. We focus on extremal multi-centered solutions, which are in the right-moving ground state, and argue that their construction reduces to a problem in Liouville theory on the disk with boundary condition which was introduced by Zamolodchikov and Zamolodchikov. In order to construct the solution one needs to solve a certain monodromy problem which we argue is solved by a vacuum classical conformal block on the sphere in a particular channel. In this way we construct multi-centered gravity solutions by using conformal blocks as a special function. We provide a check of our arguments in an example and also comment on non-extremal solutions.

Martin Kolář: *Nonlinear symmetries and gap phenomena for polynomial models in \mathbb{C}^3 .*

We give a classification of smooth real hypersurfaces of finite Catlin multitype in \mathbb{C}^3 which admit nonlinear infinitesimal CR automorphisms. The results are complete on the level of weighted homogenous polynomial models. As a consequence, we prove a sharp 1-jet determination result in the general case. As another consequence, we describe all possible dimensions of the Lie algebra of infinitesimal CR automorphisms. The result demonstrates a new gap phenomenon in dimension eight.

Ilya Kossovskiy: *On the CR-embeddability of real hypersurfaces into quadrics and CR-complexity*

A well known result of Forstneric states that most real-analytic strictly pseudoconvex hypersurfaces in complex space are not holomorphically embeddable into spheres of higher dimension. A more recent result by Forstneric states even more: most real-analytic hypersurfaces do not admit a holomorphic embedding even into a merely algebraic hypersurface of higher dimension, in particular, a hyperquadric. Explicit examples of real-analytic hypersurfaces non-embeddable into hyperquadrics were obtained by Zaitsev. In contrast, the classical theorem of Webster asserts that every real-algebraic Levi-nondegenerate hypersurface admits a transverse holomorphic embedding into a nondegenerate real hyperquadric in complex space. In this talk, I will discuss effective results on the non-embeddability of real-analytic hypersurfaces into a hyperquadric which we obtained recently with Ming Xiao. We showed that, for any $N > n \geq 1$, the defining functions $\varphi(z, \bar{z}, u)$ of all real-analytic hypersurfaces $M = \{v = \varphi(z, \bar{z}, u)\} \subset \mathbb{C}^{n+1}$ containing Levi-nondegenerate points and locally transversally holomorphically embeddable into some

hyperquadric $\mathcal{Q} \subset \mathbb{C}N + 1$ satisfy an universal algebraic partial differential equation $D(\varphi) = 0$, where the algebraic-differential operator $D = D(n, N)$ depends on n, N only. As an application, we show that for every n, N as above there exists $\mu = \mu(n, N)$ such that a Zariski generic real-analytic hypersurface $M \subset \mathbb{C}n + 1$ of degree $\geq \mu$ is not transversally holomorphically embeddable into any hyperquadric $\mathcal{Q} \subset \mathbb{C}N + 1$. We also provide an explicit upper bound for μ in terms of n, N . This gives the first effective lower bound for the CR-complexity of a Zariski generic real-algebraic hypersurface in complex space of a fixed degree.

Pavel Kočí: *Supersymmetry breaking and higher derivative terms*

In this talk I will discuss supersymmetry breaking in theories with higher derivative terms. Using superspace higher derivative terms which deform the auxiliary field potential we find that in addition to the standard supersymmetric vacuum, there is a new vacuum where supersymmetry is spontaneously broken and nonlinearly realized.

Boris Kruglikov: *Differential Invariants: calculus and count*

Consider a pseudogroup preserving a class of geometric structures. We are interested in description and counting of differential invariants of its action that is done by computing the corresponding Poincare function. The results will be illustrated by many examples.

Svatopluk Krýsl: *Symplectic dirac operator for KK-theory*

Elliptic operators, KK-theory, symplectic and metaplectic geometry – analytical aspects

Roman Lávička: *The skew Pieri rule for the symplectic group*

For the symplectic group, using a reciprocity law we obtain a skew Pieri rule, that is, a description of the irreducible decomposition of the tensor product of an irreducible representation of the symplectic group with a fundamental representation.

This is a joint work with Roger Howe, Soo Teck Lee and Vladimir Soucek.

Hong Van Le: *Cramér-Rao inequality and machine learning*

Machine learning is a subfield of computer sciences which builds models for deriving a theory from empirical data under certain computational constraints and mathematical assumptions. In my talk I shall discuss some geometric problems of machine learning and I shall report new results concerning Cramér-Rao inequality, an important geometrical tool used in machine learning, that has been recently obtained in our joint work with Jürgen Jost and Lorenz Schwachhöfer.

Jerzy Lewandowski: *Non-expanding horizons including near horizon geometries as related subject*

Non-expanding horizon (NEH) is a null, non-expanding and shear free $N-1$ dimensional surface in N dimensional spacetime. A topological assumption is the existence of compact, $N-2$ dimensional spacial slices. Geometry (q, D) of NEH consists of a degenerate metric tensor q and a torsion free covariant derivative D . The Einstein equations and energy inequalities imply constraints on the geometry. The main application of NEHs is quasi-local black hole theory. Several theorems concerning black holes have counterparts or analogies in the theory NEH geometry. Near horizon geometries (NHG) are another application of NEHs, in particular of special symmetric NEHs called extremal (degenerate) isolated horizons (IH). Solutions to Einsteins equations belonging to the Kund's class are foliated by NEHs upon the suitable topological assumption. Yet another example of NEH is conformal boundary of asymptotically flat spacetime. It's geometry describes gravitational radiation. Two intersecting NEH form a characteristic Cauchy surface for Einstein's equations called black hole holograph. Upon suitable conditions, the black hole becomes NHG. The NEH theory and applications will be presented in this lecture.

Antonella Marchesiello: *Superintegrable 3D systems in a magnetic field and Cartesian separation of variables*

We consider three dimensional superintegrable systems in a magnetic field. We study the class

of such systems which separate in Cartesian coordinates in the limit when the magnetic field vanishes, i.e. possess two second order integrals of motion of the “Cartesian type”. For such systems we look for additional integrals up to second order in momenta which make these systems minimally or maximally superintegrable and study the corresponding trajectories. We observe that the leading structure terms of the Cartesian type integrals should be considered in a more general form than for the case without magnetic field.

Michal Marvan: *Spacetimes with symmetry. From scalar invariants to exact solutions.*

Many known solutions of Einstein equations of general relativity satisfy a relation among differential invariants. E.g., the algebraically special solutions are of this kind. This idea naturally extends to space-times with symmetries, where differential invariants tend to be substantially simpler.

Alexandr Medvedev: *Sub-Riemannian Engel structures: abnormal geodesics and integrability*

We provide the first known family of examples of integrable sub-Riemannian structures admitting strictly abnormal geodesics. These examples were obtained through the analysis of the equivalence problem for sub-Riemannian Engel structures. We proved that 6 invariants define a sub-Riemannian Engel structure and described the classification of left-invariant sub-Riemannian structures of Engel type. As an application of these results we provide a criterion of strict abnormality of geodesics as well as estimates on conjugate times in terms of the obtained invariants.

Giovanni Moreno: *Second-order PDEs with large groups of symmetries*

This talk is based on a recent cooperation with D. Alekseevsky, J. Gutt and G. Manno (see <https://arxiv.org/pdf/1606.02633v1.pdf>). There the authors address the problem of constructing a special second-order (nonlinear) PDE over the adjoint contact manifold of a simple Lie group G , such that the group of (contact) symmetries of the PDE is precisely G . The so-obtained PDE is “special” because its degree, as an algebraic hypersurface in the space of the integral elements of the contact EDS, is minimal amongst all the second-order PDEs having G as their group of symmetries.

Prim Plansangkate: *Skyrmions, multi-instantons and $SU(\infty)$ -Toda equation*

We construct Skyrmions from holonomy of the spin connection of multi-Taub-NUT instantons with the centres positioned along a line in \mathbb{R}^3 . The spin connection is chosen to be in the gauge such that the resulting family of Skyrmions includes the Taub-NUT Skymion previously constructed by Dunajski. In addition, the domain of our Skyrmions is the space of orbits of the axial symmetry of the multi-Taub-NUT instantons. We obtain an expression for the induced metric on the space and its associated solution to the $SU(\infty)$ -Toda equation.

Ján Pulmann: *Minimal model of loop homotopy Lie algebras*

We show how to obtain a minimal model of a loop homotopy Lie algebra (a quantum L_∞ algebra) using the path integral in the Batalin-Vilkovisky formalism and, equivalently, via the homological perturbation lemma. This is a joint work with Martin Doubek and Branislav Jurčo.

Vladimír Souček: *Applications of the F-method in projective and Grassmannian geometry*

In recent years, branching problems for generalized Verma modules and their relation to the construction of invariant differential operators were studied in more details. A special method for computation of singular vectors in generalized Verma modules called the 'F-method' was developed and applied in some particular situations. In the lecture, some more examples in projective and Grassman geometry will be discussed. The lecture is based on the work in progress with T. Kobayashi, B. Orsted and P. Somberg.

Marek Szydłowski: *Geometry of dynamics – cosmological models as dynamical systems of Newtonian type*

The methods of dynamical systems are applied to study of dynamics of cosmological models with the R-W symmetry and barotropic equation of state. We demonstrate that there is a large class of models which dynamics reduces to the form of dynamical systems of Newtonian type. The notion of structural stability is discussed in this context.

Michal Širaň: *Integration of Courant algebroids*

A Courant algebroid can be thought of as a symplectic dg manifold of degree 2. By proving Lie's Third Theorem for L_∞ -algebroids (dg manifolds) we can integrate the underlying dg manifold of a Courant algebroid. The method is a generalization of an idea of Sullivan from Rational Homotopy Theory.

Following the AKSZ construction, one can then integrate the symplectic structure. The resulting integral is a local 2-symplectic Lie 2-groupoid. There is an analogue of Lie's Second Theorem for L_∞ -algebroids; however integration is a functor only up to homotopy. Joint work with Pavol Ševera.

Libor Šnobl: *Third-order superintegrable systems with potentials satisfying only nonlinear equations*

The conditions for superintegrable systems in two-dimensional Euclidean space admitting separation of variables in an orthogonal coordinate system and a functionally independent third-order integral are studied. It is shown that only systems that separate in subgroup type coordinates, Cartesian or polar, admit potentials that can be described in terms of nonlinear special functions. Systems separating in parabolic or elliptic coordinates are shown to have potentials with only non-movable singularities.

Dennis The: *On C-class equations*

In a short paper in 1938, Elie Cartan formulated the beautiful notion of ODE of "C-class". Moreover, Cartan gave two examples of such classes in the context of scalar ODE of 2nd order and 3rd order (considered up to point and contact transformations respectively). I'll describe the relevance of this class and discuss joint work with Andreas Cap and Boris Doubrov in which we identify C-classes for scalar and systems of higher order ODE.

Jiří Tolar: *From Lie gradings to gradings of operator algebras*

Among the gradings of simple Lie algebras over complex numbers the most important ones are the gradings by the maximal torus, also called the root or Cartan decomposition. Such a grading means a decomposition into eigenspaces of the maximal torus. The question about the existence of other fine gradings has been raised in 1989 in the seminal paper by J. Patera and H. Zassenhaus. The systematic analysis by the research group collaborating with J. Patera in Prague (M. Havlíček, E. Pelantová, M. Svobodová, J. Tolar) described besides the root grading many new gradings, especially the Pauli gradings. In quantum computing, the basic operators are the generalized Pauli matrices. They define finite quantum kinematics and generate the Pauli grading. Our general analysis of gradings of operator algebras then provides the classification of composite finite quantum kinematics.

Aleksy Tralle: *On homology Smale–Barden manifolds with K-contact and Sasakian structures*

We show that there exist 5-dimensional closed manifolds with zero first homology, which admit K-contact and do not admit Sasakian structures. Joint work with V. Munoz and J. A. Rojo.

Alena Vanžurová: *Reconstruction of an Affine Connection in Generalized Fermi Coordinates*

On a manifold with affine connection, we introduce special pre-semigeodesic charts which generalize Fermi coordinates. We use a version of the Peano’s–Picard’s–Cauchy-like Theorem on the initial values problem for systems of ODSs. In a fixed pre-semigeodesic chart of a manifold with a symmetric affine connection, we reconstruct, or construct, the connection in some neighborhood from the knowledge of the “initial values”, namely the restriction of the components of connection to a fixed $(n-1)$ -dimensional surface S and from some of the components of the curvature tensor R in the full coordinate domain. In Riemannian space, analogous methods are used to retrieve (or construct) the metric tensor of a pseudo-Riemannian manifold in a domain of semigeodesic coordinates from the known restriction of the metric to some non-isotropic hypersurface and some of the components of the curvature tensor of type $(0,4)$ in the ambient space.

Travis Willse: *Almost Einstein $(2, 3, 5)$ conformal structures*

We analyze the existence of Einstein metrics in a given conformal structure induced by a $(2, 3, 5)$ distribution via Nurowski’s Fefferman construction. Such conformal structures that admit an almost Einstein scale can be characterized in two ways: First, they are precisely the oriented conformal structures c that are induced by at least two distinct oriented $(2, 3, 5)$ distributions, in which case there is a 1-parameter family of such distributions that induce c . Second, they are characterized by the existence of a holonomy reduction to $SU(1,2)$, $SL(3,R)$, or a particular semidirect product, according to the sign of the Einstein constant of the corresponding metric.

Via the curved orbit decomposition formalism such a reduction partitions the underlying manifold into several submanifolds, each naturally endowed with a geometric structure. This establishes novel links between $(2,3,5)$ distributions and many other geometries, including: Sasaki–Einstein geometry and its paracomplex and null-complex analogues in dimension 5; Kähler–Einstein geometry and its paracomplex and null-complex analogues, Fefferman Lorentzian conformal structures, and para-Fefferman neutral conformal structures in dimension 4; CR geometry and the point geometry of second-order ordinary differential equations in dimension 3; and projective geometry in dimension 2.

Henrik Winther: *Jacobi Identity via Lie Algebra Cohomology*

Let \mathfrak{h} be a Lie algebra, and let \mathfrak{g} be an \mathfrak{h} -module with a sub-module that is equivalent to the adjoint module of \mathfrak{h} (possibly without invariant complement). We would like to classify the Lie algebra structures on \mathfrak{g} such that \mathfrak{h} is a sub-algebra and the \mathfrak{h} -module structure is the restriction of the adjoint representation of \mathfrak{g} to \mathfrak{h} . The existence of such structures leads to cohomological statements about the original module. These allow us to simplify and partially solve the Jacobi identities of \mathfrak{g} , considered as a set of algebraic equations. We also discuss applications to non-reductive Klein geometries with prescribed isotropy. Joint work with Boris Kruglikov.

Artur Woike: *New constructions of symplectically fat bundles*

It is well known that there are two general ways to endow the total space of a fiber bundle with a fiberwise symplectic form. The first one is given by the Thurston’s theorem on symplectic fibrations. The second is given by the Sternberg, Weinstein and Lerman theorems on fat bundles and symplectic manifolds. We will call the symplectic fibrations constructed with the Sternberg and Weinstein theorem *symplectically fat*.

This talk is devoted to new constructions of symplectically fat fiber bundles. The latter are constructed in two ways: using the Kirwan map and expressing the fatness condition in terms of the isotropy representation related to the G -structure over some homogeneous space.

Petr Zima: *Integrability conditions for Killing equations on homogeneous spaces*

The goal of our current work is to find interesting solutions of different types of Killing equations on specific homogeneous Riemannian (spin-) manifolds. Solving an invariant finite-type system of PDEs on homogeneous space can be reduced to purely algebraic problem by considering appropriate integrability conditions. We first discuss the problem in a general situation of arbitrary Cartan geometric structure. We show that on homogeneous spaces the integrability conditions are in fact sufficient and characterize local solutions of the original system. Then we carry out this approach explicitly in case of Killing equations. As an example we describe complete sets of solutions on Berger spheres.

Alexander Zuevsky: *Higher genus partition and correlation functions for vertex superalgebras and modular form identities*

We will give a short reminder for vertex operator algebra notion and corresponding characters. Then we discuss algebraic methods for explicit computation of the partition and correlation functions. Finally, a general way to find number theory identities for related modular forms will be given.

Vojtěch Žádník: *Tractor Frenet formulas in conformal and projective geometry*

(joint with Josef Šilhan) There are different ways to describe differential invariants of curves in regard of different geometric structures. Some standard constructions in homogeneous manifolds have very natural and very general tractorial counterparts. We reveal some details in the conformal and projective setting.