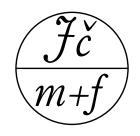
# THE 39<sup>th</sup> WINTER SCHOOL GEOMETRY AND PHYSICS

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## LIST OF PARTICIPANTS

Dmitri Alekseevsky Ľudovít Balko Temesgen Bihonegn Réamonn Ó Buachalla Andreas Čap Martin Čadek Andreas Deser Martin Doležal Maciej Dunajski Zdeněk Dušek Yaël Frégier Anton Galaev Fran Globlek Roman Golovko Jan Gregorovič Karmen Grizelj **Christian Gustad** Pavel Hájek Ondřej Hulík Denis Husadžić Goce Chadzitaskos Josef Janyška Branislav Jurčo Igor Khavkine Martin Kolář **Boris Kruglikov** Lukáš Krump Svatopluk Krýsl Radoslaw Kycia Thomas Lada Emanuele Latini Roman Lávička Van Le Andreu Llabres Tibor Macko Antonella Marchesiello Mario De Marco Martin Markl **Ruben Minasian** Rouzbeh Mohseni Jiří Nárožný

Katharina Neusser Agnieszka Niemczynowicz Jan Novák Jovana Obradović Rigel Apolonio Juarez Ojeda Pavle Pandžić Lada Peksová Tomáš Procházka Roland Púček lán Pulmann Tomáš Rusin Tomasz Rybicki Tomáš Salač Andrea Santi **Eivind Schneider** lan Slovák Vladimír Souček Martina Stojić Karen Strung Radek Suchánek losef Svoboda Pavol Ševera Josef Šilhan Mária Šimková Zoran Škoda Libor Šnobl Jan Šťovíček Dennis The Aleksy Tralle Vít Tuček **Rikard von Unge** Fridrich Valach **Orestis Vasilakis** Jakub Vošmera Jan Vysoký Henrik Winther Karolina Wojciechowicz Lenka Zalabová Petr Zima Alexander Zuevsky Vojtěch Žádník

### ANNOUNCED LECTURES

### A. INVITED LECTURES

Réamonn Ó Buachalla: Quantum Flag Manifolds: From Quantum Groups to Noncommutative Geometry Yaël Frégier: Deformation theory through examples Boris Kruglikov: Overdetermined systems of PDEs: formal theory and applications Ruben Minasian: Topics in string geometry Pavol Ševera: From braids to quantization Jan Šťovíček: Noncommutative algebraic geometry based on quantum flag manifolds

### B. Other lectures

**Dmitri Alekseevsky**: Conformal model of hypercolumns in visual cortex and its application to the visual stability problem

Ľudovít Balko: Cup-length of certain classes of flag manifolds

Andreas Čap: The bundle of Weyl structures associated to an AHS structure

Andreas Deser: Courant algebroids in the NQ-language: A case study for nilmanifolds

Maciej Dunajski: Conformally isometric embeddings.

Zdeněk Dušek: Homogeneous geodesics in Randers spaces

Anton Galaev: Comporison of two approaches to characteristic classes of foliations

Roman Golovko: On Legendrian lifts of monotone Lagrangian submanifolds

**Jan Gregorovič**: On solution of the equivalence problem for a class of 2–nondegenerate CR manifolds

**Pavel Hájek**:  $IBL_{\infty}$ -structure and string topology conjecture

Ondřej Hulík: Higher Spin Gravity and Multicentered Solutions

Denis Husadžić: Singular BGG complexes over isotropic 2-Grassmannian

Goce Chadzitaskos: Two string harmonic oscillator

**Josef Janyška**: Noether's theorem and conserved currents in Covariant Classical and Quantum Mechanics

**Igor Khavkine**: *Compatibility complexes of overdetermined PDEs of finite type, with applications to the Killing equation* 

Martin Kolář: Polynomial and rational models for real hypersurfaces in complex space Lukáš Krump: TBA

Svatopluk Krýsl: Hodge theory, associated bundles and C\*-modules

**Radoslaw Kycia**: Integrability of geodesics of totally geodesic metrics **Thomas Lada**: *TBA*  Roman Lávička: TBA Emanuele Latini: The q-linked Minkowski space Van Le: TBA **Tibor Macko**: *The higher structure sets of lens spaces* Antonella Marchesiello: Superintegrable 3D systems in a magnetic field and separation of variables Rouzbeh Mohseni: Abelian duality in two dimensions with non-trivial boundary conditions Katharina Neusser: TBA Jan Novák: Einstein meets Grothendieck: RT paradigm as Quantum Gravity Jovana Obradović: Combinatorial homotopy theory for operads Rigel Apolonio Juarez Ojeda: Homotopy theory of singular foliations Pavle Pandzić: On classification of unitary highest weight modules Tomáš Procházka: W algebras Ján Pulmann: Linear infinitesimal braidings for abelian 2-groups. Roland Púček: TBA Tomáš Rusin: On the characteristic rank and cohomolgy of oriented Grassmann manifolds Tomasz Rybicki: On the uniform perfectness and boundedness of diffeomorphism groups Tomáš Salač: TBA Andrea Santi: Killing superalgebras and high supersymmetry Martin Schnabl: On classical solutions of string field theory **Eivind Schneider**: Differential invariants of Kundt waves Jan Slovák: Traces of Tractors in Sub-Riemannian Geometry Vladimír Souček: An application of the Penrose transform for isotropic Grassmannians. Martina Stojić: Completed Hopf algebroid of formal differential operators on a Lie group Karen Strung: On C\*-algebras, dynamical systems, and classification Josef Šilhan: Symmetries of the twistor operator Mária Šimková: Are two spaces homotopy equivalent? (Algorithmic approach) Zoran Škoda: Localization approach to noncommutative flag varieties **Libor Šnobl**: Superintegrability and time – dependent integrals Aleksy Tralle: Compact Clifford-Klein forms Vit Tuček: Invariant differential operators for Hermitian symmetric spaces **Rikard von Unge**: Nonlinear realizations in partial supersymmetry breaking Fridrich Valach: Courant algebroids, Poisson-Lie T-duality and supergravity (of type II) **Orestis Vasilakis**: Multi-centered solutions in AdS3

Jakub Vošmera: Boundary states for stable branes with reduced supersymmetry on flat backgrounds Jan Vysoký: Supergravity and Poisson-Lie T-duality Henrik Winther: Quaternion-Hermitian Structures with Large Symmetry Algebra Karolina Wojciechowicz: Complete and vertical lifts of Poisson vector fields and infinitesimal deformations of Poisson tensor Lenka Zalabová: TBA Petr Zima: TBA Alexander Zuevsky: Genus two recursion formulas for correlation functions of fermionic vertex operator super algebras Vojtěch Žádník: TBA

## ABSTRACTS

**Dmitri Alekseevsky**: Conformal model of hypercolumns in visual cortex and its application to the visual stability problem

We present a model of hypercolumns as a conformasl sphere , which is a generalisation of Riemasnnian sphere miodel of Bressloff-Cowan. Simple cells in this model are parametrized by points of the Moebius group. We discuss application of this model to visual stability problem.

#### Ľudovít Balko: Cup-length of certain classes of flag manifolds

We compute the  $Z_2$  cup-length of flag manifolds F(2, 2, n) and  $F(1, 3, 2^{s+1} - 3)$  and values of the height of the third Stiefel-Whitney class of the canonical line bundle over the Grassmann manfold F(4, n).

# **Réamonn Ó Buachalla**: *Quantum Flag Manifolds: From Quantum Groups to Noncommutative Geometry*

A Drinfeld–Jimbo quantum group  $U_q(\mathfrak{g})$  is an intrinsic q-deformation of the universal enveloping algebra of a semisimple Lie algebra  $\mathfrak{g}$ , with an associated q-deformation  $\mathcal{O}_q(G)$  of the polynomial algebra of the corresponding Lie group G. Despite 30 years of intensive study, the process of *q*-deforming the differential geometry of *G* remains mysterious. In this series of lectures we present a new approach to this problem, based around a *q*-deformation of the Kähler geometry of generalised flag manifolds. Syllabus: In the first lecture we recall the basic definitions and constructions of Drinfeld–Jimbo quantum groups, and compare and contrast with classical Lie theory. We see that the definitions of parabolic and Levi subalgebras generalise directly to the noncommutative setting allowing us to construct quantum flag manifolds  $\mathcal{O}_q(G/L)$ . In the second lecture we see that every generalised quantum flag manifold of Hermitian symmetric type admits a direct *q*-deformed de Rham complex. We then show how the rich Kähler geometry of the classical setting carries over directly to the quantum case. In the final lecture, we show how this rich Kähler structure allows us to construct a spectral triple (Connes' notion of a noncommutative Riemannian spin manifold) which is a direct *q*-deformation of the Dirac–Dolbeault operators of G/L. Moreover, we see how a number of connections with other areas of mathematics naturally emerge, namely the theory of Nichols algebras, Schubert calculus, and most importantly noncommutative projective algebraic geometry, as explained in greater depth in Jan Stovicek's complementary series of lectures. (Joint work with: Petr Somberg, Jan Stovicek, Adam-Christiaan van Roosmalen)

#### Andreas Cap: The bundle of Weyl structures associated to an AHS structure

This talk reports on joint work in progress with Thomas Mettler (Frankfurt), which generalizes a construction of Dunajski and Mettler for projective structures. We work in the setting of AHS structures (parabolic geometries associated to |1|-gradings). Given such a structure on a manifold M, we construct and affine bundle  $A \rightarrow M$ , whose smooth sections are in bijective correspondence with Weyl structures for the initial AHS structure. On the manifold A, one obtains a natural almost bi-Lagrangean structure, which combines an almost symplectic structures and a pseudo-Riemannian metric of split signature and carries a natural connection. There is an efficient calculus relating this geometry to the AHS structure. Using this we show that the is symplectic iff the initial AHS structure is torsion-free and that in this case one obtains an Einstein metric. In the end of the talk, I'll outline how A can be used to study natural fully non-linear PDE associated to the AHS structure.

#### Andreas Deser: Courant algebroids in the NQ-language: A case study for nilmanifolds

Starting with Roytenberg's observation that degree-two dg symplectic manifolds are equivalent to Courant algebroids, graded manifolds with homological functions became an important tool to describe the gauge structure of certain string-inspired field theories (in particular gravity coupled to the Kalb Ramond field and double field theory). After reviewing aspects of this, I will give an application of the formalism to three-dimensional nilmanifolds equipped with an abelian gerbe structure, a prominent example in the study of T-duality.

#### Yaël Frégier: Deformation theory through examples

The aim of this mini-course is to give a gentle introduction to the main ideas behind defomation theory, in particular the algebraic tools used such as differential graded Lie algebras, operads and graded geometry.

#### Anton Galaev: Comporison of two approaches to characteristic classes of foliations

Crainic and Moerdijk defined the characteristic classes of a foliation as elements of the Čechde Rham cohomology of the leaf space; Losik defined the characteristic classes of a foliation as elements of the de Rham cohomology of the space of frames of infinite order over the leaf space. In the talk these approaches will be compared. In particular, using Losik's theory of smooth structures on leaf spaces of foliations, a new construction of the characteristic classes obtained by Crainic and Moerdijk will be given. It willbe shown that Losik's characteristic classes may be mapped to these of Crainic and Moerdijk. Also secondary classes with values in Čech-de Rham cohomology will be defined.

#### Roman Golovko: On Legendrian lifts of monotone Lagrangian submanifolds

We consider Legendrian lifts of monotone Lagrangian tori in the projective plane and relate their augmentation varieties to the corresponding Landau–Ginzburg potentials. In addition, we show that the Legendrians we get are subflexible and this leads to the refinement of the regularity conjecture of Eliashberg–Ganatra–Lazarev. This is joint work with Georgios Dimitroglou Rizell.

# **Jan Gregorovič**: On solution of the equivalence problem for a class of 2–nondegenerate CR manifolds

Let *G* be a simple Lie group, *P* a parabolic subgroup of *G* and *Q* a subgroup *P* such that P/Q is a Hermitian symmetric space. We classify the homogenous spaces G/Q that are maximally symmetric models of 2–nondegenerate CR manifolds. In particular, we solve the equivalence problem for 2–nondegenerate CR manifolds that can be modeled at each point by the space G/Q.

#### **Pavel Hájek**: $IBL_{\infty}$ -structure and string topology conjecture

The dual cyclic bar complex of the de Rham cohomology of a closed manifold M carries an  $IBL_{\infty}$ -structure which gives a chain model for the Chas-Sullivan string topology of M using the Chen's iterated integrals map. This  $IBL_{\infty}$ -structure is obtained by twisting the canonical IBL-structure with a Maurer-Cartan element constructed by picking a Green kernel as the propagator and computing integrals associated to trivalent ribbon graphs similar to those in perturbative Chern-Simons theory. I will give an overview and present some examples, observations and open questions.

#### Denis Husadžić: Singular BGG complexes over isotropic 2-Grassmannian

We construct exact sequences of invariant differential operators acting on sections of certain homogeneous vector bundles in singular infinitesimal character, over the isotropic 2–Grassmannian. This space is equal to G/P, where G is  $Sp(2n, \mathbb{C})$ , and P its standard parabolic subgroup having the Levi factor  $GL(2, \mathbb{C}) \times Sp(2n - 4, \mathbb{C})$ . The constructed sequences are analogues of the Bernstein–Gelfand–Gelfand resolutions. We do this by considering the Penrose transform over an appropriate double fibration. The result differs from the Hermitian situation.

#### Goce Chadzitaskos: Two string harmonic oscillator

We present solution of one-dimensional harmonic oscillator which obey two string forces. One for x < 0 with characteristic frequency f+ and different for x < 0 with characteristic frequency f-. The extension of this idea for the harmonic oscillator on the half line, and construction of coherent states were done. Such oscillator can be realized as a mass on the string, with second string outside or inside thefirst one, not firmly connected with the mass.

# **Josef Janyška**: Noether's theorem and conserved currents in Covariant Classical and Quantum Mechanics

Covariant Classical Mechanics and Covariant Quantum Mechanics are geometric approaches to Classical Mechanics and Quantum Mechanics on a curved spacetime fibred over absolute time and equipped with a riemannian metric on its fibres. We assume a few basic fields from which we can construct, by using covariant (natural) operations, the classical and the quantum lagrangians. By using the Noether's theorem we can classify conserved currents associated with these lagrangians.

# **Igor Khavkine**: *Compatibility complexes of overdetermined PDEs of finite type, with applications to the Killing equation*

In linearized gravity, two linearized metrics are considered gauge-equivalent,  $h_{\mu\nu} \sim h_{\mu\nu} + K_{\mu\nu}[\nu]$ , when they differ by the image of the Killing operator,  $K_{\mu\nu}[\nu] = \nabla_{\mu}v_{\nu} + \nabla_{\nu}v_{\mu}$ . A universal (or complete) compatibility operator for K is a differential operator  $K_1$  such that  $K_1 \circ K = 0$  and any other operator annihilating K must factor through  $K_1$ . The components of  $K_1$  can be interpreted as a complete (or generating) set of local gauge-invariant observables in linearized gravity. By appealing to known results in the formal theory of overdetermined PDEs of finite type and basic notions from homological algebra, we solve the problem of constructing the Killing compatibility operator  $K_1$  on an arbitrary background geometry, as well as of extending it to a full compatibility complex  $K_i$  ( $i \ge 1$ ), meaning that for each  $K_i$  the operator  $K_{i+1}$  is its universal compatibility operator.

#### Boris Kruglikov: Overdetermined systems of PDEs: formal theory and applications

In this series of lectures I will discuss formal theory of differential equations. I will start with a brief introduction to the jet-spaces and their geometry. Differential equations encoded as cofiltered submanifolds, possess rich geometric structures responsible for compatibility: symbols, curvatures and Spencer cohomology. This latter is also responsible for deformation of transitive Lie pseudogroups and I will explain this in the classical context, mentioning modifications in the filtered and super-symmetric cases. Then I will discuss how resolution theory in commutative algebra helps to construct compatibility complexes of overdetermined operators. I will finish with some applications. In particular, I will demonstrate characterization of Monge-Ampere equations and, if time permits, talk about relation to dispersionless integrability.

#### Svatopluk Krýsl: Hodge theory, associated bundles and C\*-modules

Hodge theory can be formulated in a more general framework than in that for elliptic complexes on finite rank bundles over compact manifolds. We present it for additive dagger categories (sources for TQFT-functors), and we give more specific results for pre-Hilbert spaces and almost identical to...

#### Radoslaw Kycia: Integrability of geodesics of totally geodesic metrics

Analysis of the geodesics in the space of signature (1,3) that splits in two-dimensional distributions resulting from the Weyl tensor eignespaces – hyperbolic and elliptic ones will be presented. Similar model of General Theory of Relativity coupled to Electromagnetism will be explained. Analysis of geodesic integrability will be outlined. This will be the brief overview of the manuscript [1].

Bibliography: [1] R. A. Kycia, M. Ułan, Integrability of geodesics of totally geodesic metrics, https://arxiv.org/abs/1810.00962

#### Tibor Macko: The higher structure sets of lens spaces

Joint work with L. Balko, M. Niepel and T. Rusin. We present a calculation of higher structure sets in the sense of surgery theory of lens spaces L in the cases not known before. The calculation provides us with a classification in some sense of manifolds homotopy equivalent to the product of L with a disk or a sphere. We will explain the background and related results, state the main theorem and when time permits we will also discuss main ideas of the proof.

# **Antonella Marchesiello**: Superintegrable 3D systems in a magnetic field and separation of variables

We study the problem of the classification of three dimensional superintegrable systems in a magnetic field in the case they admit integrals polynomial in the momenta, two of them in involution and at most of second order (besides the Hamiltonian). We start by considering second order integrable systems that would separate in subgroup-type coordinates in the limit when the magnetic field vanishes. We look for additional integrals which make these systems minimally or maximally superintegrable. Joint work with L. Šnobl and P. Winternitz.

#### Rouzbeh Mohseni: Abelian duality in two dimensions with non-trivial boundary conditions

In this talk, I first first give a quick review on Abelian duality based on the article written by Edward Witten, then I will Discuss the case of having a closed manifold and boundary conditions. In the end, I will give a quick glance at the Abelian T-duality in string theory.

#### Jan Novák: Einstein meets Grothendieck: RT paradigm as Quantum Gravity

Construction of a model of Quantum Gravity, which will be some day in concordance with experiments, is one of the most fascinating tasks which we have in modern theoretical physics. There are common features for all of the approaches to quantum gravity, which were developed so far. We try to review some of them briefly. We mention the non-locality, background independence and dimensional reduction. Then we introduce the concept of nonlinear graviton and we suggest the mathematical apparatus for Quantum Gravity. We claim that the fundamental apparatus is hidden in a branch of algebraic geometry, called plabic graphs. We study the bijection between decorated permutations and Le-diagrams. Then we start to investigate the connection of Grassmanians and plabic graphs. We finish with the topic of reduced plabic graphs. The interesting thing is that our approach could help to resolve the longstanding problem of mathematical formulation of Feynman's path integral. We end with an experimental evidence for our approach and we pose a list of open questions, both in mathematics and physics.

#### Jovana Obradović: Combinatorial homotopy theory for operads

I will introduce an explicit combinatorial characterization of the minimal model for the coloured operad encoding non-symmetric operads, whose structure generalizes the structure of Stasheff's associahedra operad.

#### Rigel Apolonio Juarez Ojeda: Homotopy theory of singular foliations

In this work we apply ideas from homotopy theory to the study of singular foliations via a technical lemma for left semi-model categories. When applied to the category of  $L_{\infty}$ -algebroids, this lemma enables to recover results about existence and (up to homotopy) uniqueness of universal  $L_{\infty}$ -algebroids associated to a singular foliation.

#### Pavle Pandzić: On classification of unitary highest weight modules

In this joint work with Vladimir Soucek and Vit Tucek, we redo the classification of unitary highest weight modules using only the Dirac inequality. The modules are organized in reduced translation cones over the basic cases, which range over the Hasse diagrams of Weyl group conjugates of weights given by sums of fundamental weights.

#### Ján Pulmann: Linear infinitesimal braidings for abelian 2-groups.

A symmetric, lax monoidal functor from the category of finite sets describes a higher group or groupoid, via the nerve construction. To quantize such functor, one needs to add infinitesimal braiding to the category of finite sets and extend the functor to an infinitesimally braided lax monoidal functor. This corresponds to the first order of the full quantized structure; in the case of Lie groups, the infinitesimal braiding gives a Poisson-Lie bracket. In this talk, we study an analogous problem for abelian 2-groups. Joint work with Pavol Severa.

#### Tomáš Rusin: On the characteristic rank and cohomolgy of oriented Grassmann manifolds

For the canonical k-plane bundle  $\tilde{\gamma}_{n,k}$  over the oriented Grassmann manifold  $\tilde{G}_{n,k} = SO(n)/(SO(k) \times SO(n-k))$ , the characteristic rank charrank( $\tilde{\gamma}_{n,k}$ ) measures the degree up to which the  $\mathbb{Z}_{2^{-}}$  cohomology of  $\tilde{G}_{n,k}$  is generated by Stiefel-Whitney classes  $w_i(\tilde{\gamma}_{n,k})$ . It can be used to derive upper bounds for the cup-length of  $\tilde{G}_{n,k}$ , in some cases even determining the cup-length exactly. We will present a method used to compute the characteristic rank of the canonical bundle  $\tilde{\gamma}_{n,k}$  and some applications.

#### Tomasz Rybicki: On the uniform perfectness and boundedness of diffeomorphism groups

A group is called bounded if any conjugation-invariant norm on it is bounded. Since the commutator length of a perfect group is a conjugation-invariant norm, any bounded and perfect group is uniformly perfect, i.e. every element of it can be expressed as a product of a bounded number of commutators. In the first part of this talk we review results on the boundedness and uniform perfectness of diffeomorphism groups. Next we will focus on the notion of smooth perfectness. In particular we will show how it will be interpreted in the structure of the fiber preserving diffeomorphism group of a locally trivial bundle.

#### Andrea Santi: Killing superalgebras and high supersymmetry

I will talk about joint work with José Figueroa–O'Farrill on the algebraic structure of the Lie superalgebra generated by the Killing spinors of an 11-dimensional supergravity background. I will explain that any such Killing superalgebra can be regarded as an appropriate deformation of a subalgebra of the Poincaré superalgebra and discuss applications to the classification of highly supersymmetric backgrounds. In particular, we will see that preserving more than half the supersymmetry implies the supergravity field equations. I will also elucidate the role played in this approach by a certain Spencer cohomology group, which defines the relevant notion of Killing spinor.

#### Martin Schnabl: On classical solutions of string field theory

I will review the status of classical solutions in string field theory

#### Eivind Schneider: Differential invariants of Kundt waves

Kundt waves are special pure radiation spacetimes. We give a complete description of the algebra of scalar differential G-invariants for such spacetimes in special coordinates, and discuss how differential invariants can be used to distinguish Kundt waves. Joint work with Boris Kruglikov and David McNutt.

#### Jan Slovák: Traces of Tractors in Sub-Riemannian Geometry

I will present an approach to sub-Riemannian normal geodesics motivated by tractor calculus. In particular, there are nice systems of equations coupling the fields in the sub-Riemannian distribution with fields in its annihilator, and its solutions describe the normal geodesics locally. I will try to illustrate how these equations work on some examples. This is work in progress joint with Rod Gover.

#### Vladimír Souček: An application of the Penrose transform for isotropic Grassmannians.

The Penrose transform is a versatile tool for construction of complexes of invariant differential operators on flag manifolds. Its important advantages is that it can treat both the case of regular infinitesimal characters as well as singular ones. In the lecture, an example of such application of the Penrose transform will be given for the case of maximally singular character for isotropic Grassmanians in even dimension. The lecture is based on common work with L. Krump and T. Salač.

#### Martina Stojić: Completed Hopf algebroid of formal differential operators on a Lie group

We show that the algebra Diff(e, G) of formal differential operators around the unit e of a Lie group G has the structure of a completed Heisenberg double U(g) \* #U(g) which is a completed Hopf algebroid. Here completion on the dual U(g)\* is naturally induced by the filtration on U(g). This Hopf algebroid lives internally in the symmetric monoidal category indproVect of filtered-cofiltered vector spaces, defined in my disertation, with the tensor product which is equal to the usual one between filtered vector spaces, and to the completed one between cofiltered vector spaces are dual to each other. The opposite algebra of Diff(e, G) is the noncommutative phase space  $S^{\omega} # U(g)$  from the article Lie algebra type noncommutative phase spaces are Hopf algebroids, Letters in Mathematical Physics, 107:3, 475–503 (2017).

#### Karen Strung: On C\*-algebras, dynamical systems, and classification

Interesting examples of simple C\*-algebras arise from dynamical objects such as minimal systems and mixing Smale spaces. The recent classification theorem fo C\*-algebras states that any two C\*-algebras which are simple separable unital, have finite nuclear dimension, and satisfy the UCT are classified up to isomorphism by their K-theoretic and tracial data. I will highlight the power of this theorem by showing that the C\*-algebras of minimal dynamical systems and those associated to mixing Smale spaces can both be classified by this invariant.

#### Pavol Ševera: From braids to quantization

The aim of these lectures is to show how deformation quantization works in simple cases, and to explain the background material of braid groups, braided monoidal categories, and Drinfeld associators. We shall quantize Poisson-Lie groups (to Hopf algebras) and, more generally, moduli spaces of flat connections on surfaces with decorated boundaries.

#### Zoran Škoda: Localization approach to noncommutative flag varieties

It is common to look for coset spaces of Hopf algebras by consideration of coinvariants for coactions of quotient Hopf algebras. Usually those are too small, and one may need to localize

in a way compatible with coaction to ensure sufficiently many coinvariants to describe the quantum coset spaces. In a work with G. Bohm, we have proved when such (and much more general) coset spaces provide noncommutative schemes, with principal examples yielding a construction of quantum group flag varieties. I shall sketch the example of what I call the universal noncommutative flag varieties which contain the quantum flag varieties as their small subvarieties but which do not require q-commutation relations, but are closer to certain Cohn localizations of free associative algebras. This example is fundamental in providing a geometric interpretations of the quasideterminant calculus of Gelfand and Retakh, and also in going beyond the flatness assumptions which are crucial in the work with Bohm.

#### Libor Šnobl: Superintegrability and time - dependent integrals

While looking for additional integrals of motion of several minimally superintegrable systems in static electric and magnetic fields, we have realized that in some cases Lie point symmetries of Euler-Lagrange equations imply existence of explicitly time-dependent integrals of motion through Noether's theorem. These integrals allow a completely algebraic determination of the trajectories (including their time dependence) although the systems don't exhibit maximal super-integrability in the usual sense. Report on work in progress, in collaboration with my student Ondřej Kubů.

#### Jan Šťovíček: Noncommutative algebraic geometry based on quantum flag manifolds

We present an approach to noncommutative algebraic geometry which focuses on the study of abelian categories, viewing them as "categories of coherent sheaves on a non-commutative variety". We show how to obtain such abelian categories from a differential calculus on quantum flag manifolds, which are constructed from Drinfeld-Jimbo guantum groups. Sylabus: Classical algebraic geometry is based on the study of commutative rings, categories of modules over them, or sheaves of such rings and modules. Typically, the rings are those of polynomial functions on an affine variety and modules are for instance the collections of global vector fields. In projective geometry, one often lacks, in analogy to complex analysis, interesting global functions and vector fields. This forces one to consider sheaves of local sections. We explain the necessary formalism which leads to the abelian category of coherent sheaves and its derived category. As classical quantum flag manifolds are based on algebraic groups, we need to understand the translation of the group structure to the algebraic language. This leads to the concept of Hopf algebras, various constructions with them and their homological properties. This is in fact also the key structure carried by the Drinfeld-Jimbo guantum groups and will be explained in the second lecture. Finally, we construct a quantum deformation of the category of coherent sheaves using a non-commutative version of the Dolbeault algebra. We show that using the non-commutative differential calculus from Réamonn Ó Buachalla's lectures and some representation theory of quantum groups, our categories of sheaves on quantum projective spaces possess some key properties of the classical categories of coherent sheaves (Serre duality, a tilting object). This is an account on a joint work with Réamonn Ó Buachalla and Adam-Christiaan van Roosmalen.

#### Aleksy Tralle: Compact Clifford-Klein forms

I will present my recent results on the compact Clifford-Klein forms problem.

#### Vit Tuček: Invariant differential operators for Hermitian symmetric spaces

Hermitian symmetric spaces G/K are Riemannian symmetric spaces for which there exists invariant complex structure. As complex manifolds they are isomorphic to  $G^{\mathbb{C}}/P$  where P is a parabolic subgroup of  $G^{\mathbb{C}}$  whose reductive part is  $K^{\mathbb{C}}$ . Any  $G^{\mathbb{C}}$ -invariant differential operator acting on sections of associated bundle  $G^{\mathbb{C}} \times_P \mathbb{F}_{\lambda}$  is given by singular vector in Verma module  $\mathcal{U}() \otimes_{\mathcal{U}(\mathcal{Y})} \mathbb{F}^*_{\lambda}$ . These singular vectors can be described by polynomial solutions to a system of PDEs. Using some classical results in representations theory of classical simple Lie algebras, one needs only elementary calculations in multivariable calculus to obtain all singular vectors in scalar Verma modules. This gives all invariant differential operators acting on line bundles.

#### Fridrich Valach: Courant algebroids, Poisson-Lie T-duality and supergravity (of type II)

We give a reformulation of the generalized Ricci tensor and scalar curvature on Courant algebroids. This allows us to easily prove (in the general setup, including the case of the so called 'dressing cosets') the compatibility of the Poisson-Lie T-duality and the string background equations. Moreover, using this framework, we obtain new solutions to modified supergravity equations on symmetric spaces. Both, the generalized Ricci tensor and scalar curvature, can also be obtained from the variation of a natural action functional.

#### **Jakub Vošmera**: Boundary states for stable branes with reduced supersymmetry on flat backgrounds

We present exact expressions for consistent elementary boundary states in type II superstring compatified on special 4-tori describing branes preserving less than 16 supercharges. While being manifestly superconformal, these boundary states are shown to violate all possible linear gluing conditions on the bosonic and fermionic worldsheet oscillators along internal directions of the compactification 4-tori. Our calculation proceeds by recasting the  $\mathcal{N} = (2, 2)$  worldsheet sigma model on these 4-tori in terms of  $\mathcal{N} = 2$  minimal models, along the lines of Gepner's construction. Imposing general permutation gluing conditions on the  $\mathcal{N} = (2, 2)$  generators is shown to yield various unstable and stable branes, where the stable ones include the known 1/4-BPS bound states of D*p*-branes as well as previously unconsidered non-BPS branes, which do not couple to massless RR sector.

#### Jan Vysoký: Supergravity and Poisson-Lie T-duality

Courant algebroids can be used to geometrically describe a plurality between a certain class of two-dimensional sigma models targetted in homogeneous spaces of Lie group pairs. Corresponding supergravity theories (low-energy effective actions) should reflect this construction. This is proved using Courant algebroid connections, leading to systems of algebraic equations. We give some examples of solutions of beta equations.

#### Henrik Winther: Quaternion-Hermitian Structures with Large Symmetry Algebra

We consider the geometric implications of high symmetry dimension for almost quaternion– Hermitian structures, a generalization of the quaternion–Kähler condition. In particular we obtain the submaximal symmetry dimension and the symmetry gap, and classify all models which admit these symmetry dimensions. We identify models which are quaternion Kähler, locally conformally quaternion–Kähler, and quaternion Kähler with torsion.

# **Karolina Wojciechowicz**: Complete and vertical lifts of Poisson vector fields and infinitesimal deformations of Poisson tensor

In this talk it will be showed that both complete and vertical lifts of a Poisson vector field from a Poisson manifold  $(M, \pi)$  to its tangent bundle  $(TM, \pi_{TM})$  are also Poisson. This fact will be used to describe the infinitesimal deformations of Poisson tensor  $\pi_{TM}$ . Some of their properties will also be studied and there will be presented an extensive set of examples in a low dimensional case.

Alexander Zuevsky: Genus two recursion formulas for correlation functions of fermionic vertex

## operator super algebras

We report on new results concerning genus two recursion formulas for correlation functions of fermionic vertex operator super algebras and their applications.

# GENERAL INFORMATION

39<sup>th</sup> WINTER SCHOOL GEOMETRY AND PHYSICS Srní, Czech Republic January 12–19, 2019 http://conference.math.muni.cz/srni/

### Contact

Geometry and Physics 2019 Department of Mathematics Faculty of Sciences – Masaryk University Kotlářská 2 611 37 Brno Czech Republic

## srni@math.muni.cz

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