Special Session of the Central European Seminar
In Celebration of Peter Michor's 75th Birthday

Masaryk University, Brno, Czech Republic

June 6 – June 8, 2024

Schedule

Thursday, June 6, 2024

- **14:00 – 14:40** Andreas Cap
  Partial G-structures and partial Cartan geometries.
- **15:00 – 15:40** Armin Rainer
  On the semialgebraic Whitney extension problem
- **16:00 – 16:40** Stefan Haller
  Regularized determinants of the Rumin complex on nilmanifolds with (2,3,5) geometry

Friday, June 7, 2024

- **10:00 – 10:40** Simon Hochgerner
  Valuation of long term guarantees
- **11:00 – 11:40** Josef Teichmann
  On the relation of real analytic functions on path spaces and signature expansions
- **14:00 – 14:40** Martin Bauer
  Geometric Analysis of the Generalized Surface Quasi-Geostrophic Equations
- **15:00 – 15:40** Philipp Harms
  Geometric perspectives on neural networks
- **16:00 – 16:40** Konstanze Rietsch
  Totally positive Toeplitz matrices and a tropical Edrei theorem

Saturday, June 8, 2024

- **9:00 – 9:40** Cornelia Vizman
  Foliations on codimension one nonlinear Grassmannians
- **10:00 – 10:40** Martins Bruveris
  Bootstrapping confidence bands for ROC curves
- **11:00 – 11:40** Jan Slovak
  Notes on curved translation principle illustrated on the Grassmannian geometries.

Abstracts

Martin Bauer
Title: Geometric Analysis of the Generalized Surface Quasi-Geostrophic Equations
Abstract: In this talk we are interested in the geometry of the beta-SQG equations, a family of PDEs in two dimensions which interpolate between the Euler equations of ideal hydrodynamics and the inviscid surface quasi-geostrophic equation. It has been recently observed that these equations can be derived as Euler-Arnold equations on the group of symplectiomorphisms and In this talk we will analyze the properties of this geometric interpretation. In the first part we will study the induced geodesic distance, which is related to the variational formulation of the PDE and in the second part we will show precisely when the corresponding Riemannian exponential map is non-linear Fredholm of index 0. Finally, we will further illustrate this by examining the distribution of conjugate points via a Morse theoretic approach. This talk is based on work with Philipp Harms, Patrick Heslin, Gerard Misiołek and Stephen C. Preston.

Martins Bruveris
Title: Bootstrapping confidence bands for ROC curves
Abstract: ROC curves are a key tool to understand the performance of binary classifiers in machine learning. As with all measurements, there is uncertainty and confidence bands, i.e., confidence intervals for curves, can be used to measure this uncertainty. In this talk I will propose a bootstrapping-based method to compute confidence bands, test its coverage properties and show how to compute it efficiently, even for larger datasets.

Andreas Cap
Title: Partial G-structures and partial Cartan geometries.“
Abstract: I will discuss generalizations of the concepts of G-structures and of Cartan geometries, which locally can be interpreted as describing smooth families of ordinary G-structures and Cartan geometries, respectively. It turns out that appropriate constructions of canonical Cartan connections generalize to this setting. I will discuss this for the example of AHS-structures.

Stefan Haller
Title: Regularized determinants of the Rumin complex on nilmanifolds with (2,3,5) geometry
Abstract: A generic rank two distribution in dimension five is a tangent 2-plane field on a 5-manifold which is maximally non-integrable, i.e., with growth vector (2,3,5). These intriguing distributions can equivalently be described as parabolic geometries associated with the exceptional Lie group G2. In this talk we will focus on spectral properties of the Rumin complex associated with a (2,3,5) distribution. This is a hypoelliptic complex of natural, higher order differential operators that computes the de Rham cohomology. We will discuss recent results on zeta regularized determinants of this complex on nilmanifolds.

Philipp Harms
Title: Geometric perspectives on neural networks
Abstract: Why are trained neural networks typically ‘nice’ functions, particularly in over-parameterized regimes? How can neural networks be scaled up in size yet remain trainable? How does the choice of network architecture impact the training dynamics? Riemannian geometry on infinite-dimensional manifolds provides a unifying framework for investigating these topics both geometrically and analytically. I will present some old and new approaches in these directions.

Simon Hochgerner
Title: Valuation of long term guarantees
Abstract: In the context of life insurance with profit participation, the future discretionary benefits (FDB), which are a central item for Solvency II reporting, are generally calculated by computationally expensive Monte Carlo algorithms. We derive analytic formulas to estimate lower and upper bounds for the FDB. The method is designed for real world applications, and examples involving public data from Germany and anonymized reporting data from Austria will be provided in the talk. (Joint work with Florian Gach, Eva Kienbacher and Gabriel Schachinger)

Armin Rainer
Title: On the semialgebraic Whitney extension problem
Abstract: In 1934, Whitney raised the question of how one can decide whether a function f defined on a closed subset X of $\mathbb{R}^n$ is the restriction of a C^m function on $\mathbb{R}^n$. He gave a characterization in dimension $n = 1$. The problem was fully solved by Fefferman in 2006. In this talk, I will discuss a related conjecture: if a semialgebraic function $f : X \to \mathbb{R}$ has a C^m extension to $\mathbb{R}^n$, then it has a semialgebraic C^m extension. In particular, I will show that the $C^{1,\omega}$ case of the conjecture is true in a uniformly bounded way, for each semialgebraic modulus of continuity ω. The proof is based on the existence of semialgebraic Lipschitz selections for certain affine-set valued maps and on a uniform semialgebraic version of Whitney's extension theorem. This is joint work with Adam Parusinski.

Konstanze Rietsch
Title: Totally positive Toeplitz matrices and a tropical Edrei theorem
Abstract: The classical Edrei theorem from the 1950's gives a parametrisation for infinite upper-triangular totally positive real Toeplitz matrices by pairs of sequences of positive real parameters with finite sum. These infinite matrices (and their parameters) are central for understanding characters of the infinite symmetric group, as was discovered by Thoma, who reproved Edrei's theorem in the 1960's. There is also a totally different (totally positive) theorem about Toeplitz matrices that relates to quantum cohomology of flag varieties and mirror symmetry [R,06]. Namely, this theorem provides an (inverse) parametrization in terms of 'quantum parameters' for the finite Toeplitz matrix case. This talk will give an overview of these two theories and introduce new tropical versions of them. Toeplitz matrices in the tropical world turn out to have a nice combinatorial description. We also uncover a surprising relationship between the classical Edrei parameters and the quantum parameters of quantum cohomology. This work builds on results of Judd and Ludenbach and relates also to Lusztig's parametrisation of his canonical basis.

Jan Slovak
Title: Notes on curved translation principle illustrated on the Grassmannian geometries
Abstract: The Janzen-Zuckermann translation principle proved to be a great tool to understand the structure of homomorphisms of generalized Verma modules, and thus the understanding of the linear invariant differential operators on the corresponding Klein's models. This is all based on the fact that the Verma modules are in fact topological duals of the modules of jets of sections of the relevant homogeneous vector bundles. The purely algebraic generalization bring the semiholomic Verma modules based on the semi-holonomic jets and their link to curved Cartan geometries was clarified in the old paper by Mike Eastwood and myself aiming at the example of conformal Riemannian geometry, more than 25 years back (Journal of Algebra, 1997). I shall remind those concepts and tools in general, talking about Grasmannian geometries. The talk is based on a work in progress with Vladimir Soucek (where we focus on the particular case of the (3,3)-Grassmannians).

Josef Teichmann
Title: On the relation of real analytic functions on path spaces and signature expansions.
Abstract: Signature expansions and associated kernel techniques are important tools in approximation theory on path spaces. We show some relations to more classical notions of real analytic functions on path spaces and to some concepts of invariant theory. We then head towards a better understanding of reproducing kernel Hilbert spaces related to signature kernels. (joint works with Christa Cuchiero, Walter Schachermayer, and Valentin Tissot-Daguette).

Cornelia Vizman
Title: Foliations on codimension one nonlinear Grassmannians
Abstract: A volume form gives rise to integrable distributions of finite codimension on the nonlinear Grassmannian of submanifolds of codimension one. These are the isodrastic distribution (analogous to Weinstein's isodrastic distribution for symplectic manifolds) and the isovolume distribution which contains it. Their leaves are useful in the description of coadjoint orbits of the (exact) volume preserving diffeomorphism group (obtained via a new dual pair). Joint work with Stefan Haller.