Toward the Effective ∞-Topos

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Abstract.

As an elementary topos, Hyland's effective topos $\mathcal{E}ff$ models extensional Martin-Löf type theory with an impredicative universe of propositions [1]. But it also contains another impredicative universe of so-called "modest sets" which is *not* a poset [2]—a fascinating combination not consistent with classical foundations. In joint work with Frey and Speight [3], we proposed a higher-dimensional version of this model with a *univalent* impredicative universe and used it to give new impredicative encodings of some (higher) inductive types. This model was based on cubical assemblies and exploited the constructive character of the recently introduced cubical Quillen model structures [4, 5]. As was subsequently shown, however, the subtopos of 0-types in this model was not equivalent to $\mathcal{E}ff$, but to a larger realizability topos.

In addition to containing a non-degenerate, impredicative, univalent universe, the elementary ∞-topos $\mathcal{E}ff_\infty$ should include $\mathcal{E}ff$ as its subtopos of 0-types. This will provide an example of a non-Grothendieck elementary ∞-topos. As a candidate for $\mathcal{E}ff_\infty$ we here propose an ∞-category of *coherent stacks* over the regular category of assemblies. We show that $\mathcal{E}ff_\infty$ is locally cartesian closed as an ∞-category and that its subcategory of 0-types is indeed $\mathcal{E}ff$.

This is joint work with Mathieu Anel and Reid Barton, building on prior joint work with Jacopo Emmenegger and Pino Rosolini [6].

References

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