Lax Monoidal Structures from Monoidal Structures

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Abstract.

We show that for any full subcategory $\mathcal{C}$ of a (unbiased) monoidal category $\mathcal{D}$, the presheaf category $\mathsf{Set}^{\mathcal{C}^{op}}$ inherits a (unbiased) lax monoidal structure. Our proof, which uses [1], establishes a result of independent interest: that $\mathcal{C}$ inherits a lax promonoidal structure [2]. We then use a generalization of Day convolution to transport this structure into the presheaf category. We discuss examples of lax monoidal structures that arise this way: the cartesian product on simplicial sets [3], the Boardman-Vogt tensor product on dendroidal sets [7], and the Gray tensor product on Θ_2-sets [6]. The first of these is monoidal while the latter two are lax. Future work will study conditions under which the lax monoidal structure constructed via this method are homotopical in the sense of [4], which can be thought of as presenting a monoidal structure in the ∞-categorical sense [5].

References

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