Naturality in Weak ω-categories
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Abstract. Coherence data in a weak higher category, such as associators and unitors, are expected to be natural in their arguments. We show that this is the case of globular weak ω-categories in the sense of Leinster [5]. Explicitly, we produce fillers of naturality squares for every operation in the theory of ω-categories. An application of this work is the construction of cones and cylinders over globular pasting diagrams, as well as their binary compositions, analogous to the work of Lanari [4] for ω-groupoids. Similar to the strict case [3], we expect this to be a step towards a model structure on the category of weak ω-categories. We have implemented our construction in the proof assistant Catt [2] for ω-categories, allowing us to export our proofs to Hott [1].

References

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