Lax idempotent monads in homotopy theory

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Abstract.
Lax idempotent monads, also known as Kock–Zöberlein monads, are monads T on a 2-category such that the unit $\eta_T\colon T \Rightarrow TT$ is a fully faithful right adjoint of the multiplication $\mu\colon TT \Rightarrow T$. A typical example is, for a fixed class of colimits, the functor $T\colon \mathcal{C}\mathrm{at} \to \mathcal{C}\mathrm{at}$ that sends a category to its free cocompletion with respect to these colimits.

In this talk, I will describe an extension of the theory of lax idempotent monads to the setting of $(\infty, 2)$-categories. Many facts about lax idempotent monads on 2-categories have analogues for lax idempotent monads on $(\infty, 2)$-categories—in particular, they can be constructed from very minimal data in such a way that all higher coherences "come for free". This is an important feature in $(\infty, 2)$-category theory, since it is generally impossible to construct such coherences "by hand".

What makes this extension desirable is that many important categories in homotopy theory can be described as categories of (co)algebras over an (op)lax idempotent (co)monad. Examples include the $(\infty, 2)$-category of cocomplete $(\infty, 1)$-categories, the $(\infty, 2)$-category of ∞-operads and the $(\infty, 2)$-category of compactly assembled categories, the latter of which plays a crucial role in Efimov's groundbreaking discovery of continuous K-theory. I will discuss some of these examples in detail and show that several important properties of these $(\infty, 2)$-categories are immediate consequences of the theory of lax idempotent monads.