# Compactological Spaces: Constructing condensed mathematics from classical topology

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**Abstract**.

The theory of condensed mathematics was introduced by Peter Scholze and Dustin Clausen in a sequence of lecture notes ([1, 2]) from 2019 onwards, in part to introduce and work with an abelian replacement for the famously not-so-well-behaved categories of topological abelian groups or more generally topological modules. Among many other concepts [1, 2] introduce quasiseperated condensed sets which serve as a class of non-degenerate and topologically seperated condensed sets. Scholze and Clausen remark that quasiseperated condensed sets correspond to compactological spaces, as introduced by Lucien Waelbroeck from 1967 onwards ([3, 4]) using the language of point-set topology.

We make this relationship explicit and demonstrate that and how it is canonical. Moreover, we prove that condensed sets form the universal ex/reg completion of the regular category of compactological spaces to a Barr-exact category (and Grothendieck topos). This relation carries over to compactological algebraic objects where the description of the completion simplifies drastically. We conclude by demonstrating that condensed modules are given, up to equivalence, by "formal fractions" of their compactological counterpart.

Our investigation, based on a forthcoming paper, does not only respond to the question of the relationship between quasiseperated and all condensed objects, it also permits an accessible description of condensed objects for those without prior contact with algebraic geometry, sheaves or topoi.


## References


[1] D. Clausen, and P. Scholze, *Condensed Mathematics*, Lecture Notes (2019).

[2] D. Clausen, and P. Scholze, *Condensed Mathematics and Complex Geometry*, Lecture Notes (2022).

[3] L. Waelbroeck, *Some theorems about bounded structures*, Journal of Functional Analysis 1.4, 1967.

[4] L. Waelbroeck, *Topological Vector Spaces and Algebras*, Lecture Notes in Mathematics, 1971.