Gray products of diagrammatic (∞, n)-categories

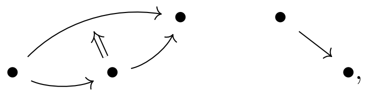
C. Chanavat

Clémence Chanavat (clemence.chanavat@taltech.ee)
Tallinn University of Technology

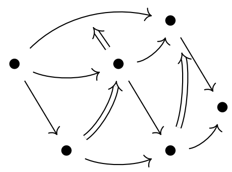
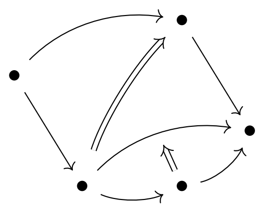
Amar Hadzihasanovic (amar.hadzihasanovic@taltech.ee)
Tallinn University of Technology

Abstract.

The Gray tensor product serves as a directed refinement of the cartesian product in higher categorical settings and is particularly useful to model all sorts of (co)lax constructions. In this talk, we present diagrammatic sets, home of the diagrammatic model of (∞, n)-categories [1], and their interaction with the Gray tensor product. Diagrammatic sets are sheaves over a category of combinatorial pasting diagrams, known as *regular directed complexes*, and main subject of Hadzihasanovic's recent monograph [2], where it is shown that regular directed complexes are monoidal with respect to the Gray product $- \otimes -$. Pictorially, we can represent the Gray product of the 2-cell and the 1-cell



respectively, as a "right cylinder" 3-cell with input and output 2-boundaries

 and  respectively.

After presenting some of the features of this construction, like its interaction with the pasting of pasting diagrams, or its distributivity under duals, we extend the Gray product to diagrammatic sets via the theory of Day convolution, and study some of its notable properties, following the work in progress [3]. For one, distinguishing the Gray tensor product from the cartesian product, we consider its interaction with higher invertible *round diagrams* (read *cells*), known as equivalences: *if u is an equivalence in X **or** v is an equivalence in Y, then $u \otimes v$ is an equivalence in $X \otimes Y$.* For another, we demonstrate that the Gray product preserves ω-equivalences, that is, morphisms $f\colon X \to Y$ of diagrammatic sets inducing an essential surjection hom-wise $f\colon X(u,v) \to Y(f(u), f(v))$ for all parallel pairs u, v. This result is of homotopical importance: ω-equivalences are precisely the weak equivalences of diagrammatic (∞, n)-categories for the model structure described in [1].

References

[1] C. Chanavat, A. Hadzihasanovic, Model structures for diagrammatic (∞, n)-categories, preprint arXiv:2410.19053v2, 2024.

[2] A. Hadzihasanovic, Combinatorics of higher-categorical diagrams, preprint arXiv:2404.07273v2, 2024.

[3] C. Chanavat, The diagrammatic (∞, n)-model structures are monoidal, to appear, 2025.