

Closed categories, pro-operads and Goodwillie calculus

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Abstract.

Eilenberg and Kelly [EK66] introduced the notion of a closed, not necessarily monoidal, structure on a category. We show that the category of pro-objects in a closed category $\mathcal{V}$ inherits a closed structure, which is not typically monoidal even when $\mathcal{V}$ is. Our strategy is to embed $\mathrm{Pro}(\mathcal{V})$ into the opposite of the category of $\mathcal{V}$ -enriched endofunctors on $\mathcal{V}$.

Our motivation is the case where $\mathcal{V}$ is the category of symmetric sequences with (right-)closed monoidal structure given by the composition product, and our application is to Goodwillie’s functor calculus. Using previous work with Arone [AC15, AC16], we show that the Goodwillie-Taylor towers of spectrum-valued functors on a pointed ∞ -category $\mathcal{C}$ are classified by right modules over a ‘pro-operad’, that is, a monoid in the closed structure on pro-symmetric sequences. That pro-operad is an example of an ‘endomorphism pro-operad’ constructed from a certain inverse sequence of functors from $\mathcal{C}$ to spectra.

References

- [AC15] Gregory Arone and Michael Ching, *A classification of taylor towers of functors of spaces and spectra*, *Advances in Mathematics* **272** (2015), no. 0, 471 – 552.
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- [EK66] Samuel Eilenberg and G. Max Kelly, *Closed categories*, *Proc. Conf. Categorical Algebra* (1966), pp. 421–562.