Projective crossed modules in semi-abelian categories

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Abstract.

We characterize the projective objects in the category of internal crossed modules [5, 4] within any (Janelidze–Márki–Tholen [6]) semi-abelian category. When this category forms a variety of algebras, the internal crossed modules again constitute a semi-abelian variety, ensuring the existence of free objects, and thus of enough projectives. We show that such a variety is not necessarily Schreier—subojects of free objects are again free—, but does satisfy the so-called Condition (P) [3]—meaning the class of projectives is closed under protosplit subobjects—if and only if the base variety satisfies this condition. As a consequence, the non-additive left chain-derived functors of the connected components functor are well defined (in the sense of [3]) in this context.

The main references of this talk are [2, 1].

References

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