

The monad of factorizations and its decomposition

Arnaud Duvieusart

Arnaud Duvieusart (arnaud.duvieusart@uclouvain.be)
UCLouvain - FNRS

Abstract.

Given a short exact sequence $K \xrightarrow{k} X \xrightarrow{q} Q$ in a semi-abelian category $\mathcal{C}$, and a morphism $\varphi: K \rightarrow L$, one can show [1] that φ can be extended to a morphism

$$\begin{array}{ccccc} K & \xrightarrow{k} & X & \xrightarrow{q} & Q \\ \varphi \downarrow & & \downarrow f & & \parallel \\ L & \xrightarrow{l} & Y & \xrightarrow{q'} & Q \end{array} \quad (1)$$

of short exact sequences if there exists an action $\xi_L^X: X \bowtie L \rightarrow L$ of X on L such that φ is a morphism of X -actions $(K, \chi_K^X) \rightarrow (L, \xi_L^X)$ and

$$(\varphi \rtimes X)^* \chi_L^{L \rtimes X} = [k, 1]^* \xi_L^X. \quad (2)$$

In this talk, we give a new interpretation of this result and explore some generalizations and applications. To this end, we consider the category $\mathcal{C}^q$ whose objects are morphisms (1) for a fixed regular epimorphism q , which may be seen as a co-slice of the category of short exact sequences in $\mathcal{C}$. Taking the object L yields a functor $\mathcal{C}^q \rightarrow \mathcal{C}$. We show that this functor is monadic.

As particular cases of monads T^q induced by an adjunction of this type, we obtain the monads of internal actions $L \mapsto X \bowtie L$, as well as the monads of the form $X \mapsto A + X$ for a fixed object A . Furthermore, we show that the functor part of such a monad admits a natural decomposition as a semidirect product. Using this fact, we show that its algebras can similarly be decomposed into an action $\xi_L^X: X \bowtie L \rightarrow L$ and a morphism $\varphi: K \rightarrow L$. We then show that the pairs (ξ_L^X, φ) that give rise to a T^q -algebra are precisely those satisfying the equivariance condition and (2).

We will then explain how such monads can be used to describe higher order semidirect products (as defined by Carrasco and Cegarra [2, 3]) in semi-abelian categories.

References

- [1] A. S. Cigoli, S. Mantovani, G. Metere, *A push forward construction and the comprehensive factorization for internal crossed modules*, Appl. Categ. Structures 22 (2014), no. 5-6, 931–960
- [2] P. Carrasco, A. M. Cegarra, *Group-theoretic algebraic models for homotopy types*, J. Pure Appl. Algebra 75 (1991), No. 3, 195–235
- [3] *A Dold-Kan theorem for simplicial Lie algebras*, Theory Appl. Categ. 32 (2017), 1165–1212