

Confluence of Term Rewriting Systems with Variable Binding

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Abstract. We consider the confluence of term rewriting systems for syntax with variable binding. Our contribution is a generalised confluence theorem in the spirit of Aczel [1] in the context of the model of second-order abstract syntax in the object-classifier topos [3, 2].

We recall from [3, 2] that a signature with variable-binding operators Σ induces a strong term monad T_Σ on $\mathbf{Set}^{\mathbb{F}}$, for $\mathbb{F}$ the category of finite cardinals and functions. We define a rewrite rule to be given by a pair of terms in $T_\Sigma(M)_0$ for a presheaf of meta-variables M . A rewrite rule ρ inductively defines a reduction relation $\sim_\rho \subseteq T_\Sigma(0) \times T_\Sigma(0)$ in $\mathbf{Set}^{\mathbb{F}}$. Closely following Aczel’s approach, we present a coherence property for a rewrite rule ρ that is shown to yield the confluence of the reduction relation $\sim_\rho$. As an application, we examine the confluence of β -reduction in the λ -calculus.

References

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- [3] M. Fiore, G. Plotkin, and D. Turi, *Abstract Syntax and Variable Binding*, In Proceedings of the 14th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS 1999), pages 193–202. IEEE Computer Society, 1999.