Well-pointed endofunctors on $(\infty, 1)$-categories

S. Henry

Simon Henry (Shenry2@uottawa.ca)
University of Ottawa

Abstract. (based on a joint work with Mathieu Anel). Well-pointed endofunctors are a tool developed by Kelly in the 80s to formalize most of the "transfinite iterative construction" we encounter in category theory. That is, construction of new objects in a category by repeating a given process an infinite number of times, until it hopefully converges. This includes, for example, colimits in the category of algebras for a monad, free algebras over endofunctors and pointed endofunctors, free monoids, formally inverting an element in a commutative monoid, etc...

In this talk I will present how this theory generalizes to the setting of $(\infty, 1)$-categories, though most of the interesting phenomena are happening at the level of 2-categories, so no knowledge of ∞-category theory is strictly required.

Most of the time, generalizing a result from 1-category theory to $(\infty, 1)$-category theory is relatively straightforward: Once the 1-category theoretic result is presented in a sufficiently nice way, we can just translate it to the ∞-categorical context, to obtain a formally very similar result, by just replacing the basic category theory results used in the proof with their higher categorical analogues. In the rare case where one of the basic results hasn't been established for higher categories yet, we need to establish it, generally using a model-dependent argument.

This talk is about a case where things did not work like this at all: the theory of well-pointed endofunctors on higher categories turn out to look quite different from its 1-categorical counterpart: In the higher categorical setting of the theory, there is an additional obstruction to the convergence of the iteration that appears at the level of 2-cells, and is relatedto braid groups.