Topoi of Automata

R. Hora

Ryuya Hora (hora@ms.u-tokyo.ac.jp)
The University of Tokyo

Abstract.

In this talk, based on [2] and ongoing work, we explore a topos-theoretic perspective on automata theory and regular languages.

We begin with the naive presheaf topos Σ-Set, the category of presheaves on the free monoid generated by a fixed alphabet Σ. The stalk functor at the canonical point of Σ-Set reveals its connection to the coalgebraic and functorial [1] approaches to automata theory.

Next, we construct a Grothendieck topos $\Sigma\text{-Set}_{\text{o.f.}}$ that fully encodes the structure of regular languages. We then explain how this topos arises naturally from four distinct perspectives on regular language theory: deterministic finite automata (DFA), finite monoids, the Myhill–Nerode theorem, and profinite topology.

If time allows, we show how hyperconnected geometric morphisms from $\Sigma\text{-Set}_{\text{o.f.}}$ describe the syntactic monoid of a language, and argue that many "geometric invariants" remain to be uncovered in this context.

References

[1] Thomas Colcombet and Daniela Petrisan. *Automata minimization: a functorial approach*, Logical Methods in Computer Science 16, 2020.

[2] R. Hora, Topoi of automata I: Four topoi of automata and regular languages, preprint arXiv:2411.06358, 2024.