Bicolimit Presentations of Type Theories

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Abstract. Presentations of algebras are a way of defining algebras using generators and relations that these generators satisfy. A more categorical point of view is that every algebra can be expressed as a coequaliser of free algebras. In this talk, we explore how similar ideas can be used in semantics of type theory.

In his PhD thesis [1], Taichi Uemura proposed a definition of type theories that allows functorial semantics of type theory: a type theory is a small category with finite limits and a class of arrows satisfying some properties. Models are then functors preserving all the structure.

We examine the 2-categorical aspects of this approach. Using tools developed in [2] and [3], we

- define a notion of signature that allows us to freely generate type theories by some simpler data (this is formalised as an existence of a free-forgetful biadjunction);
- explain that type theories can be glued together via bicolimits.

Luckily, (a big class of) bicolimits and the 'free generation' interact well with our notion of semantics: Whenever we have a bicolimit of freely generated type theories (i.e. if we have a bicolimit presentation of a type theory), we can understand its semantics as long as we understand the semantics of the generating data.

In this way, we can construct type theories whose semantics recovers various flavours of semantics of dependent type theory. For example, we can create a type theory whose models are natural models [4] or a type theory whose models are natural models with Π-types.

References

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