Representables in fuzzy category theory

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Abstract.

Fuzzy category theory describes category-like structures in which potential objects and potential morphisms are respectively objects and morphisms only to a certain degree [1]. By easing the requirements on objects and morphisms, we can create models for situations that cannot be directly described using the tools of classical or enriched category theory. This approach enables the study of structures with varying degrees of existence, offering a localized perspective on objects and morphisms while maintaining an otherwise context-free setting.

Enriching categories over the symmetric monoidal closed category of L-sets [2], we effectively assign each morphism in a category a membership degree from a given lattice L. Since L-set enriched categories naturally embed into fuzzy categories—where both objects and morphisms can be assigned membership degrees—this motivates the exploration of representations arising from fuzzy functors. By examining fuzzy functors mapping into the fuzzy category $\boldsymbol{FL}$-**Set**, we extend the classical theory of representable functors into the realm of fuzzy categories.

In this talk, we establish conditions under which such fuzzy functors can be considered representable, particularly in the context of threshold categories, which are obtained by fixing an idempotent element in the underlying quantale structure. We analyze the relationship between fuzzy functor representations and threshold categories, demonstrating that representable fuzzy functors naturally induce representable enriched $\boldsymbol{L}$-**Set**-functors when viewed within an enriched categorical framework.

References

[1] A. Šostak, *On a concept of a fuzzy category*, In: 14th Linz Seminar on Fuzzy Set Theory: Non-Classical Logics and Their Applications, Linz, Austria, 1992, 63 – 66.

[2] A. Pultr. *On categories over the closed categories of fuzzy sets*, In: Abstracta. 4th Winter School on Abstract Analysis, Zdeněk Frolík (Ed.), Czechoslovak Academy of Sciences, Praha, 1976, 47 – 63.