The formal theory of vector fields for tangentads

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Tangent category theory, as introduced by Rosický [5] and later revisited by Cockett and Cruttwell [1], is a well-established categorical framework for differential geometry. In [4], a formal approach was adopted to provide a genuine Grothendieck construction in the context of tangent category theory, by introducing **tangentads**, previously named tangent objects. A tangentad is to a tangent category as a formal monad [6] is to a monad on a category. Since tangent category theory captures important geometric notions such as vector fields, it is natural to wonder whether or not these constructions can be lifted to the formal context. In particular, one wonders how to formalize important concepts such as vector fields for tangentads.

In this talk, I discuss the formal notion of tangentads. We present numerous examples of tangentads, such as (split) restriction tangent categories [1], tangent fibrations [2], and tangent monads [3]. I also introduce a formal construction for vector fields on tangentads by highlighting their universal property for tangent categories. I show how to recover the usual operations between vector fields, such as the Lie bracket. Moreover, I prove the existence of vector fields in the presence of PIE limits.

The paper can be found at https://arxiv.org/abs/2503.18354.

References

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