

Differential Graded Algebras in Differential Categories

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Abstract.

Differential categories [1] provide a categorical framework for the algebraic foundations of differentiation and also provide the categorical semantics of Differential Linear Logic [2]. There are many interesting examples of differential categories that are based on differentiating various kinds of interesting functions, such as polynomials, power series, smooth functions, etc. Differential categories have been quite successful in formalizing various important concepts related to differentiation such as derivations and Kähler differentials [3], differential algebras [4], de Rham cohomology [6], antiderivatives [5], etc. Following this line of work, in this talk, I will explain how to formalize differential graded algebras in a differential category. I will first provide a friendly introduction to the world of differential categories, leading us to the definition of differential graded algebras relative to a differential category. In models based on polynomial differentiation, we recover the usual commutative differential graded algebras, while in models based on differentiation smooth functions, we recover differential graded C^∞ -rings [7]. To further justify our definition, we will explain how the monad of a differential category can be lifted to its category of chain complexes, and how the algebras of the lifted monad correspond precisely to differential algebras of the base category, with the free algebras given by the de Rham complexes. Moreover, it turns out that the category of chain complexes of a differential category is again a differential category, pointing us towards a possible motivating example of differential dg-categories. This talk is based on joint work with Chiara Sava.

References

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