

Extensional concepts in intensional type theory, revisited

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Abstract.

In his Ph.D. dissertation, Hofmann [Hof95] constructs an interpretation of extensional type theory in intensional type theory, subsequently proving a *conservativity* result of the former over the latter extended by functional extensionality and uniqueness of identity proofs. Interestingly, Hofmann’s proof is “stronger” than the statement of his theorem, as the requisite language in which to speak of conservativity and equivalence of dependent type theories did not exist at the time.

In [KL18], it was observed that the category of models of a dependent type theory carries the structure of a left semi-model category. This structure was used by Isaev [Isa18] to define a *Morita equivalence* of dependent type theories as a translation between them that induces a Quillen equivalence between their categories of models.

This talk is based on our recent paper [KL25], in which we give a direct proof of Morita equivalence between the extensional type theory and the intensional type theory extended functional extensionality and UIP. While Hofmann proves that the initial models of these theories are suitably equivalent, we generalize this result to all possible extensions of the base theories by types and terms, including propositional equalities.

Therefore, thanks to proving Morita equivalence, one does not need to prove an analogue of Hofmann’s result for any new extension but instead appeal to our result addressing all extensions once and for all. As new variants and extensions of intensional type theory are constantly proposed, this reduces the burden of proving their expected properties by making what should be formal formal.

References

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