Giry monad revisited

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Abstract.

Grasping probability by means of category theory is a longstanding problem. In 1981 Giry [1] suggested two versions of a monad structure to capture random processes—one on the general category of measurable spaces, the other on the category of Polish spaces. More recently, Tobias Fritz [2] came up with the concept of Markov categories. Unfortunately, the latter does not work together with the traditional set-theoretic way to model probability theory. And the former approach is quite weak in way of practical utility for general measurable spaces, while Polish spaces do not cover all reasonable applications.

In my contribution I want to revisit Giry's former approach. The enterprise is worthwhile, as actually a more granular understanding of the particularities of set-theoretic probability was only achieved after Giry wrote her paper—culminating for instance in [3, 4, 5].

At the outset, I am going to explain how the actual issue is the monad multiplication—based on a counter example by Ramachandran [6]. To overcome this obstacle, I, firstly, continue by visiting set theories close to ZFC. Afterwards, I will consider the Giry construction merely as an endofunctor. In this set up, I will present a new result of weak pullback preservation for the general class of countably separated measurable spaces with the Giry monad restricted to measures with quotient regular conditional probability property. Limit preservation properties (for directed limits) had already served Giry as a touchstone.

References

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