Bicategories of lax fractions

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Abstract.

The well-known calculus of fractions of Gabriel and Zisman [1] provides a convenient way to formally invert morphisms in a category. This was extended to 2-categories by Pronk in [2]. On the other hand, in [3] the second author developed a calculus of lax fractions for order-enriched categories that formally turns a given class of morphisms into left adjoint right inverses (and provides fine control over when the Beck–Chevalley condition holds). We extend this construction to general 2-categories.

One application is a construction of the bicategory of (strict) monoidal categories and lax monoidal functors from the 2-category of strict monoidal categories and strict monoidal functors by formally adding left adjoints to the morphisms whose underlying functors are fully faithful right adjoints.

The data for the construction involves the original 2-category $\mathcal{X}$ and a collection Σ of squares, commuting up to isomorphism, whose horizontal morphisms are to become left adjoint right inverses, and which will themselves will become Beck–Chevalley squares. This collection of squares must stastify a number of axioms. The resulting bicategory of lax fractions $\mathcal{X}[\Sigma_*]$ will have the same objects as $\mathcal{X}$ and 1-cells given by cospans $A \xrightarrow{f} I \xleftarrow{s} B$ where s is a horizontal morphism from Σ. The 2-cells are certain equivalence classes of diagrams of the form

$$\begin{tikzcd}
A \arrow[r, "f"] \arrow[dd, equal] & I \arrow[d, "x_1"'] \arrow[dr, phantom, "\sum^{\delta_1}"] & B \arrow[l, "r"'] \arrow[dd, equal] \\
 & X \arrow[dl, Rightarrow, "\alpha"'] & B \arrow[l, "x_3"'] \\
A \arrow[r, "g"'] & J \arrow[u, "x_2"] \arrow[ur, phantom, "\sum^{\delta_2}"] & B, \arrow[l, "s"]
\end{tikzcd}$$

where Σ denotes that the squares come from the collection Σ.

This bicategory is universal in the sense that, for each bicategory $\mathcal{Y}$, there is an biequivalence between the bicategory of pseudofunctors from $\mathcal{X}[\Sigma_*]$ to $\mathcal{Y}$ and a bicategory whose objects are pseudofunctors from $\mathcal{X}$ to $\mathcal{Y}$ that send the horizontal morphisms of Σ to left adjoint right inverses and the squares of Σ to Beck–Chevalley squares.

References

[1] P. Gabriel and M. Zisman, *Calculus of Fractions and Homotopy Theory*, Ergebnisse der Mathematik und ihrer Grenzgebiete, 2. Folge, vol. 35, Springer, 1967.

[2] D. A. Pronk. *Etendues and stacks as bicategories of fractions*, Compos. Math. 102 (1996), no. 3, 243–303.

[3] L. Sousa. *A calculus of lax fractions.* J. Pure Appl. Algebra 221 (2017), no. 2, 422–448.