The cohomology objects of a semi-abelian variety are small

S. Mattenet

Sébastien Mattenet (sebastien.mattenet@uclouvain.be)
UCLouvain

Tim Van der Linden (tim.vanderlinden@uclouvain.be)
UCLouvain

Raphaël Jungers (raphael.jungers@uclouvain.be)
UCLouvain

Abstract.

A well-known, but often ignored issue in Yoneda-style definitions of cohomology objects via collections of n-step extensions (i.e., equivalence classes of exact sequences of a given length n between two given objects, usually subject to further criteria, and equipped with some algebraic structure) is, whether such a collection of extensions forms a set. We explain that in the context of a semi-abelian variety of algebras, the answer to this question is, essentially, yes: for the collection of all n-step extensions between any two objects, a set of representing extensions can be chosen, so that the collection of extensions is "small" in the sense that a bijection to a set exists.

We further consider some variations on this result, involving double extensions and crossed extensions (in the context of a semi-abelian variety), and Schreier extensions (in the category of monoids).

References

[1] S. Mattenet, T. Van der Linden and R. Jungers, The cohomology objects of a semi-abelian variety are small, preprint arXiv:2411.17200, 2025.

[2] D. Bourn, and G. Janelidze,*Charaterization of protomodular varieties of universal algebras*,Theory Appl. Categ. 11 (2003), no. 6, 143–147.

[3] J. Huebschmann,*Crossed n-fold extensions of groups and cohomology*, Comment. Math. Helv. 55 (1980), 302–314.

[4] L. Rédei,*Die Verallgemeinerung der Schreierschen Erweiterungstheorie*, Acta Sci. Math. (Szeged) 14 (1952), 252–273.

[5] D. Rodelo and T. Van der Linden,*Higher central extensions and cohomology*, Adv. Math. 287 (2016), 31–108.

[6] Ch. A. Weibel,*An introduction to homological algebra*, Cambridge Stud. Adv. Math., vol. 38, Cambridge Univ. Press, 1994.