

2-dimensional commutativity and Fox's theorem: sketchy approach

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Abstract. There is a notion of commutativity for any Lawvere theory T : as an example, the theory for commutative monoids is commutative and the one for arbitrary ones is not. On the other side of the monad-theory correspondence, we have a corresponding definition of a commutative monad which asks for a commutativity of a certain diagram. In the 2-dimensional setting, we can define a pseudocommutativity structure on a monad by wanting certain diagram to commute up to a coherent isomorphism – but the corresponding notion for Lawvere theories has not been considered.

In this talk, I am going to define a pseudocommutativity structure on a Lawvere 2-theory T using the Gray tensor product. Seeing 2-theories as enriched sketches, we can show that 2-category of T -models in $\mathbf{Cat}$ admits a symmetric multicategory structure which is closed. We use this observation to prove a far-reaching generalization of Fox's theorem which in its classical form says that for any symmetric monoidal category C , the coproduct in $\mathbf{CMon}(C)$, the category of commutative monoids in C , coincides with a tensor product in C .