Unified approach to pointfree T_0-spaces

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Abstract.

The classical correspondence between topological spaces and frames $\Omega\colon \mathsf{Top}^{\mathsf{op}} \leftrightarrows \mathsf{Frm}\colon \mathsf{pt}$ restricts to a dual equivalence between *sober spaces* and *spatial locales*. Thus, sober spaces are fully embedded in the category of frames, permitting us to study such topological spaces via algebraic/order-theoretical techniques.

Another correspondece that allows the study of a certain class of spaces via pointfree techniques is T_D-duality [1], which captures a correspondence between the category T_D-spaces and the category of frames with D-homomorphisms, obtained by suitably adjusting the spectrum functor. Since not every sober space is a T_D-space, this allows for a class of non-sober spaces to be studied pointfreely; however, since not every T_D-space is sober, these two approaches appear incompatible, despite being dual in some sense.

Inspired by Raney duality [2], which describes a correspondence between T_0-spaces and Raney algebras, Suarez [5, 6] developed a pointfree account of this duality via *Raney extensions* of frames, which allows for a pointfree description of all T_0-spaces, which include both sober and T_D-spaces. In fact, the spectra for both the T_D-duality and the classical correspondence appear as special cases of this pointfree Raney duality, by considering the smallest and the largest Raney extensions of a frame L.

Other pointfree notions of T_0-space are given by the *McKinsey-Tarski algebras* [3] and *strictly zero-dimensional biframes* [4], both of which contain Raney extensions as a distinguished subclass.

Aiming to understand the relationship between these three different notions of pointfree T_0-spaces, we introduce and study an abstract notion of T_0*-extensions of frames*, which are given by suitable functors $\mathcal{O}\colon \mathsf{C} \to \mathsf{Frm}$ that extend the classical correspondence $\Omega\colon \mathsf{Top}^{\mathsf{op}} \leftrightarrows \mathsf{Frm}\colon \mathsf{pt}$ to capture all T_0-spaces. By studying the fibers of $\mathcal{O}$, we are able to study various aspects of T_0-spaces abstractly.

References

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