Ord-Mal'tsev categories

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Abstract.

The aim of this talk is to explore the 1-dimensional algebraic Mal'tsev property from an Ord-enriched point of view. A 1-dimensional (regular) Mal'tsev category [2, 1] may be characterised through nice properties on (internal) relations such as:

- every reflexive relation $R\colon X \nrightarrow X$ is an equivalence relation;
- any relation $D\colon X \nrightarrow Y$ is *difunctional*, meaning that $DD^{\circ}D \subseteq D$.

The proof of such characterisations are easily obtained through the calculus of relations, which has been well established for regular categories for several years (see [1]).

In order to explore the Mal'tsev property in an Ord-enriched context we have to develop the calculus of relations for regular Ord-categories. We adapt the calculus of relations given in [4], which was done for regular Pos-categories. To capture the enriched features of a regular Ord-category and obtain a good calculus, the relations we work with are precisely the *order ideals*. We introduce the notion of Ord-*Mal'tsev category* and show that these may be characterised through enriched versions of the above mentioned properties adapted to order ideals. Any Ord-enrichment of a 1-dimensional Mal'tsev category is necessarily an Ord-Mal'tsev category. We also give some examples of categories which are not Mal'tsev categories, but are Ord-Mal'tsev categories (see [3]).

This talk is based on joint work with Maria Manuel Clementino [3].

References

[1] A. Carboni, G. M. Kelly, M. C. Pedicchio, *Some remarks on Maltsev and Goursat categories*, Appl. Categ. Struct. **14** (1993) 385–421.

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[3] M.M. Clementino, D. Rodelo, *Enriched aspects of calculus of relations and 2-permutability*, J. Algebra Appl. (to appear); DMUC preprint 24-33 (2024) 24 pgs.

[4] V. Aravantinos-Sotiropoulos, *The exact completion for regular categories enriched in posets*, J. Pure Appl. Algebra 226(7) (2022) 106885.