Stability from the categorical point of view

J. Rosický

Jiří Rosický (rosicky@math.muni.cz)
Masaryk university

Abstract.

Stability theory is a thriving modern branch of model theory, initiated by Morley [7] and largely developed by S. Shelah [8]. It aims to classify structures based upon their logical complexity and its central tool is that of independence relations, basic examples of which include linear independence in vector spaces and algebraic independence in fields. In a series of papers [3, 4, 5], we have shown that it can be viewed as categories equipped with a class of commuting squares, called independent squares. The resulting stable independence is closely related to cofibrant generations of morphisms. Later, we extended this approach to unstable independences in [1, 2].

We are going to survey this approach and to supplement it by new results. In particular, we will show how the stability spectrum is related to the small object argument. This makes it possible to treat superstability in acts over monoids (in [6] was done for modules over a ring).

References

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