

Double functorial representation of indexed monoidal structures

J. Siqueira

José Siqueira (`jose@topos.institute`)
Topos Institute

Abstract.

The data of a logical doctrine acts in two directions: that of the *substitution* operation, and that of *quantification*. Double Category Theory provides the tools to capture these two actions and their relationship concomitantly: in a double category of spans, re-indexing and acting on predicates can be packaged as tight and loose arrows respectively, and in there tight arrows always have conjoints. By mapping these conjoints into a double category of quintets, we obtain adjunctions internal to a 2-category (classically, of posets). Moreover, by using the notion of adequate triple, we can study situations where the indexing category only has certain pullbacks.

Adding a monoidal structure to the fibers (and the double functor) allows us to capture both the Beck-Chevalley and Frobenius conditions, and the talk will explore this idea in more detail. We will concern ourselves with the monoidal analogues of regular hyperdoctrines and similar structures (where the objects of predicates are not necessarily posets, but rather live in some 2-category), and show how they are equivalent to (lax symmetric monoidal) double functors between spans and quintet double categories. If time permits, I will mention the connection between these ideas, Double Categorical Systems Theory, and assume-guarantee reasoning.

References

- [1] Grandis, M. Limits in double categories. *Cahiers de Topologie et Géométrie Différentielle Catégoriques*. **40** pp. 162-220 (1999).
- [2] Shulman, M. Framed bicategories and monoidal fibrations. *Theory and Applications of Categories*. **20** pp. 650-738 (2008).
- [3] Moeller, V. Monoidal Grothendieck Construction. *Theory and Applications of Categories*. **35**, 1159-1207 (2020).
- [4] Haugseng, R., Hebestreit, F., Linskens, S. & Nuiten, J. Two-variable fibrations, factorisation systems and ∞ -categories of spans. (2023). [arXiv:2011.11042](https://arxiv.org/abs/2011.11042).
- [5] Paré, R. Retrocells. *Theory and Applications of Categories*. **40**, 130-179 (2024).