When is $\mathsf{Cat}(\mathcal{Q})$ cartesian closed?

I. Stubbe

Isar Stubbe (isar.stubbe@univ-littoral.fr)
Université du Littoral, France

Abstract.

Let $\mathcal{Q}$ be any (small) quantaloid (= category enriched in sup-lattices). Viewed as a bicategory, it serves as base for enriched categories and functors, thus producing the category $\mathsf{Cat}(\mathcal{Q})$. The category of ordered sets is an example of this construction (taking $\mathcal{Q}$ to be the one-object suspension of the two-element boolean algebra), as is the category of generalized metric spaces (taking $\mathcal{Q}$ to be the Lawvere quantale of positive real numbers). Yet these two examples behave quite differently: the first is cartesian closed, whereas the second is not [1]. This raises the question: can we find necessary and/or sufficient conditions on $\mathcal{Q}$ to have $\mathsf{Cat}(\mathcal{Q})$ cartesian closed? A result in [2] shows exactly how the exponentiability of each individual $\mathcal{Q}$-category depends on $\mathcal{Q}$. In this talk, we use this to give an elementary characterization of those quantaloids $\mathcal{Q}$ for which $\mathsf{Cat}(\mathcal{Q})$ is cartesian closed. With this characterization, we unify several known cases (previously proven using *ad hoc* methods) and we give some new examples. Based on joint work with Junche Yu [3].

References

[1] M. M. Clementino and D. Hofmann, *Exponentiation in $\mathcal{V}$-categories*, Topology Appl. **153** (2006) pp. 3113–3128.

[2] M. M. Clementino, D. Hofmann and I. Stubbe, *Exponentiable functors between quantaloid-enriched categories*, Appl. Categor. Struct. **17** (2009) pp. 91–101.

[3] I. Stubbe and J. Yu, When is $\mathsf{Cat}(\mathcal{Q})$ cartesian closed?, preprint arXiv:2501.03942, 2025.