

Extensive Morphisms

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Abstract.

Extensivity of a category can be described as a property of coproducts in the category, namely, that they are disjoint and universal. An alternative perspective is that it is a property of morphisms in a category. This talk explores this viewpoint by introducing natural notions of extensive and coextensive morphisms. A category is (co)extensive if and only if each morphism in the category is (co)extensive. In addition, these notions allow for the study of (co)extensivity in categories in which only some of the morphisms are (co)extensive. For instance, while extensivity trivializes pointed categories, morphisms with trivial kernels in the category of pointed sets are extensive. We examine various examples of (co)extensive morphisms and discuss the implications of (co)extensivity within certain classes of morphisms.

For example, in Universal Algebra, the strict refinement property and Fraser-Horn property can be characterized by the coextensivity of product projections and of surjective homomorphisms, respectively, allowing for a categorical generalization of these concepts. Additionally, the coextensivity of all split monomorphisms in a Barr-exact category implies coextensivity of the entire category.

References

- [1] E. Theart and M. Hoefnagel, On extensivity of morphisms, preprint arXiv:2502.12695, 2025.