A Kaluzhnin–Krasner embedding theorem for non-associative algebras?

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Abstract.

For (non-abelian) groups A and B, the *Universal Embedding Theorem* of Kaluzhnin and Krasner [6] says that the *(unrestricted) wreath product* $A \wr B$ acts as a universal receptacle for any group G viewed as an extension from A to B. Recently, versions of the result were obtained for Lie algebras [7] and cocommutative Hopf algebras [1].

The aim of this talk is to report on joint work with Bo Shan Deval and Xabier García-Martínez [2], where we attempt to prove a version of the theorem for general varieties of non-associative algebras over a field. In such a variety, the objects are vector space equipped with a bilinear multiplication, eventually subject to a set of polynomial identities. By establishing a connection with the concept of *local algebraic cartesian closedness* [4], we find a *universal Kaluzhnin–Krasner embedding theorem*, valid in semi-abelian categories [5]. Via the results of [3], this allows us to fully characterise those varieties of non-associative algebras over an infinite field where the embedding theorem holds.

References

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