

The commuting tensor product of multicategories

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Abstract.

Given two algebraic theories S and T , their commuting tensor product $S \otimes T$ is a new algebraic theory in which the operations of S and T are required to commute with each other. The analogue of this operation for symmetric operads is the famous Boardman-Vogt tensor product. A unified account of these commuting tensor products was given in [4].

Here, we extend this analysis in two respects. First, we generalise it so as to make it applicable to the many-object case, recovering as special case the Boardman-Vogt tensor product of symmetric multicategories, subsuming work of Elmendorf and Mandell [2]. Secondly, we investigate how the commuting tensor product acts on bimodules, with the goal of generalising work of Dwyer and Hess on the Boardman-Vogt on bimodules between symmetric operads [1] to the setting of [5].

This work is carried out in the context of a double category equipped with an oplax monoidal structure, in the sense of [3], considering monads and monad multimorphisms therein, and involves showing that these form a representable multicategory. The application to the Boardman-Vogt tensor product arises by considering the double category of coloured symmetric sequences with the arithmetic product of [3].

References

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