# Immersions, Submersions, Local Diffeomorphisms, and Relative Cotangent Complexes in Tangent Categories

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**Abstract**.

The theory of tangent category theory is an area of study which exploded since it was discovered just over a decade ago (in its modern framework) in [1] by Cockett and Cruttwell. Tangent categories themselves are an abstract categorical framework to study differential-geoemtric reasoning by making use of the semantic and structural relations that the tangent bundle monad on the category **SMan** of smooth (real) manifolds encodes. This perspective has allowed for tangent categories to be used in theoretic computer science, differential geometry, applied category theory, (differential) linear logic, and even in algebraic geometry.

An important theme in tangent category theory (and various geometric flavours of mathematics) is to introduce and abstract the core structures of the morphisms which are important for differential geometry. For instance, both [3] and [2] the authors have abstracted, characterized, and studied how to incarnate submersions and local diffeomorphisms in the tangent categorical world. This begs the question how to incarnate other classes of maps such as immersions, unramified maps, and more into the general tangent category world.

In this talk based on joint work with JS Lemay (for which a preprint will appear on the arXiv between now and CT 2025), I will indicate some of the work we have done in this direction. I will introduce the tangent-categorical definitions of immersions, unramified maps, submersions, and local diffeomorphisms. Additionally, I will characterize how each class of maps incarnates in the tangent categories **SMan** of smooth manifolds, of commutative algebras $\mathbf{CAlg}_R$ over a commutative rig (semiring) $R$, the opposite category of commutative algebras $\mathbf{CAlg}_R^{\mathrm{op}}$ over a commutative rig $R$, and the category $\mathbf{Sch}_{/S}$ of schemes over a base scheme $S$. In particular, I will indicate that submersions (and local diffeomorphisms) in the tangent category of affine schemes involve classes of morphisms *more* general than formally smooth morphisms (and formally étale morphisms, respectively). Finally, I will also indicate how to define the relative cotangent complex in a tangent category and how it arises in many examples of interest.

## References

[1] J. R. B. Cockett and G. S. H. Cruttwell, *Differential structure, tangent structure, and SDG*, Appl. Categ. Structures 22 (2014), no. 2, 331–417.

[2] G. S. H. Cruttwell and M. Lanfranchi, *Pullbacks in tangent categories and tangent display maps*, arXiv Preprint 2502.20699. 2025.

[3] B. MacAdam, *Vector bundles and differential bundles in the category of smooth manifolds*, Appl. Categ. Structures 29 (2021), no. 2, 285–310.