Infinite and Non-Rigid Reconstruction Theory

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Abstract.
Monadic reconstruction theory—relating additional structure of a monad to structure on its Eilenberg–Moore category—can be seen as a generalisation of classical Tannaka–Krein duality, in which one reconstructs a compact group from its category of representations. Results of this kind were obtained for oplax monoidal monads—also called bimonads—by Moerdijk [1] and McCrudden [2]; for Hopf monads by Bruguières, Lack, and Virelizier [4, 5]; and for $*$-autonomous and linearly distributive monads by Pastro and Street [6, 7]. Crucial in all of these statements is the involvement of a fibre functor, which generalises the, classically, forgetful strict monoidal functor to the category of vector spaces or bimodules over a ring.

A different kind of reconstruction is possible if one foregoes such a fibre functor. For example, given a monoidal category $\mathcal{C}$, one could ask when a given $\mathcal{C}$-module category $\mathcal{M}$ is equivalent to the Eilenberg–Moore category of some monad on $\mathcal{C}$. That is, we only recover the algebraic object of interest up to Morita equivalence.

This talk generalises such a reconstruction result by Ostrik [3] about Hopf algebras on finite tensor categories to the general case of right exact lax module monads on a nice module category over a general abelian monoidal category with enough projectives. Crucially, the proof does not need any rigidity assumptions on the underlying category, and in fact leads to a characterisation of right exact lax module monads up to Morita equivalence.

As an application, we give conceptual proofs of the fundamental theorem of Hopf modules, and the fact that a bimonad is Hopf if and only if it is strong as a module monad over its base category. The talk is based on joint work with Mateusz Stroiński [8].

References

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