July 17th, 2025

CT2025, Brno (CZ)

2-classifiers for 2-algebras

in 7 minutes

Matteo Capucci
University of Strathclyde / ARIA Creator

David Jaz Myers
Topos Research UK

What is Set?

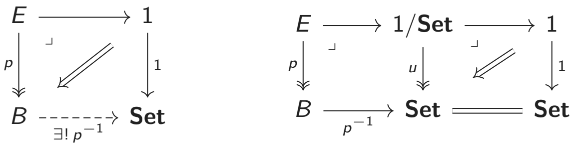
What is Set?

In **Cat**, it can be characterized internally as the **discrete opfibration 2-classifier**:

$$\begin{array}{ccc} E & \longrightarrow & 1 \\ {\scriptstyle p}\downarrow & \lrcorner \quad \swarrow & \downarrow{\scriptstyle 1} \\ B & \underset{\exists!\, p^{-1}}{\dashrightarrow} & \mathbf{Set} \end{array}$$

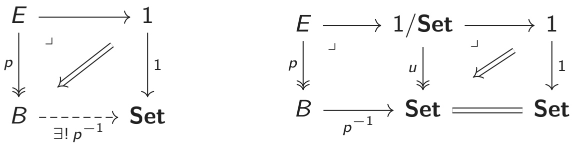
What is Set?

In **Cat**, it can be characterized internally as the **discrete opfibration 2-classifier**:



What is Set?

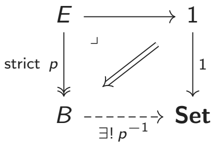
In **Cat**, it can be characterized internally as the **discrete opfibration 2-classifier**:



What happens when we equip categories with structure? Is **Set** still the 2-classifier?

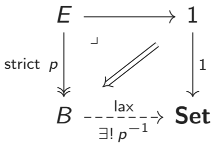
What is Set?

e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



What is Set?

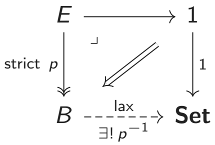
e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

What is Set?

e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



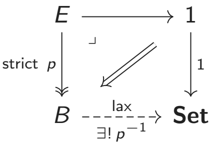
What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

Let T = free symmetric monoidal category 2-monad, consider

$$T\mathbf{Set} \dashrightarrow \mathbf{Set}$$

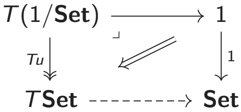
What is Set?

e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



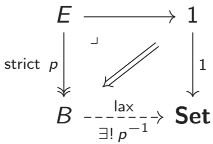
What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

Let T = free symmetric monoidal category 2-monad, consider



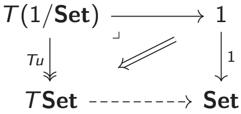
What is Set?

e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

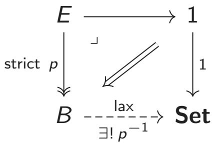
Let T = free symmetric monoidal category 2-monad, consider



$(X_i)_{i \leq n}$

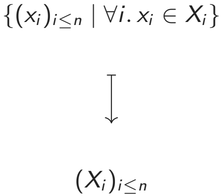
What is Set?

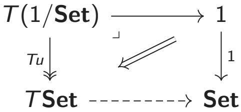
e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

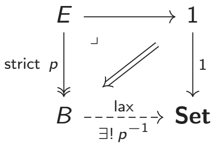
Let T = free symmetric monoidal category 2-monad, consider





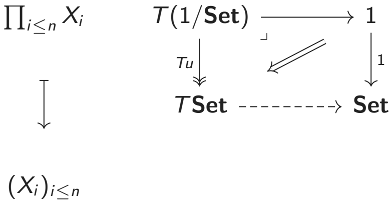
What is **Set**?

e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



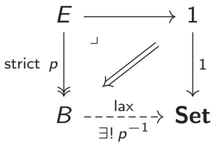
What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

Let T = free symmetric monoidal category 2-monad, consider



What is Set?

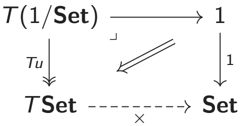
e.g. consider monoidal categories, and let's try to classify **strict monoidal** discrete opfibrations:



$$\overline{p^{-1}} : E_b \times E_{b'} \to E_{b \times b'}$$

What is the monoidal structure on **Set**? What about the laxator of p^{-1}?

Let T = free symmetric monoidal category 2-monad, consider



$$(X_i)_{i \leq n} \quad \longmapsto \quad \prod_{i \leq n} X_i$$

Sketch of the main result

Theorem

If T is a nice enough 2-monad on a nice enough 2-category $\mathbb{K}$ with 2-classifier $1 \xrightarrow{\tau} \Omega$, then

Sketch of the main result

Theorem

If T is a nice enough 2-monad on a nice enough 2-category $\mathbb{K}$ with 2-classifier $1 \xrightarrow{\tau} \Omega$, then

- *Ω is a strict T-algebra, via the map classifying Tu:*

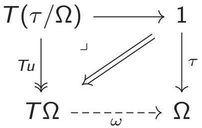
$$\begin{array}{ccc} T(\tau/\Omega) & \longrightarrow & 1 \\ {\scriptstyle Tu}\big\downarrow & \swarrow & \big\downarrow{\scriptstyle \tau} \\ T\Omega & \underset{\omega}{\dashrightarrow} & \Omega \end{array}$$

Sketch of the main result

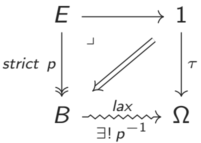
Theorem

If T is a nice enough 2-monad on a nice enough 2-category $\mathbb{K}$ with 2-classifier $1 \xrightarrow{\tau} \Omega$, then

- *Ω is a strict T-algebra, via the map classifying Tu:*



- *$1 \xrightarrow{\tau} (\Omega, \omega)$ 2-classifies strict discrete opfibrations in $\mathbb{A}\mathbf{lg}_{\mathrm{lx}}(T)$.*

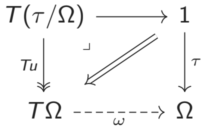


Sketch of the main result

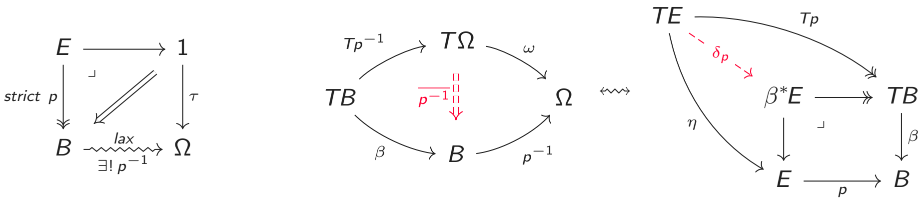
Theorem

If T is a nice enough 2-monad on a nice enough 2-category $\mathbb{K}$ with 2-classifier $1 \xrightarrow{\tau} \Omega$, then

- *Ω is a strict T-algebra, via the map classifying Tu:*



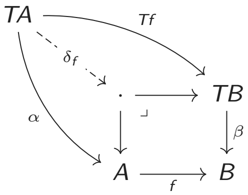
- *$1 \xrightarrow{\tau} (\Omega, \omega)$ 2-classifies strict discrete opfibrations in $\mathbb{A}\mathbf{lg}_{\mathrm{lx}}(T)$.*



Cartesian maps of T-algebras

Definition

Let $f : A \to B$ be a strict T-morphism. We call **T-cartesianity defect of f** the canonical comparison map induced by the pullback below:

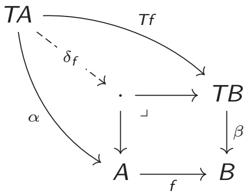


We say f is **T-cartesian** when its defect is invertible.

Cartesian maps of T-algebras

Definition

Let $f : A \to B$ be a strict T-morphism. We call T-**cartesianity defect of** f the canonical comparison map induced by the pullback below:



We say f is T-**cartesian** when its defect is invertible.

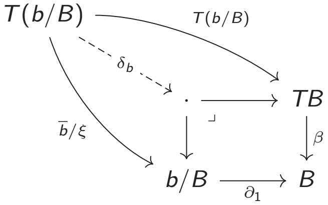
Corollary

p^{-1} is T-strong if and only if p is T-cartesian.

Cartesian objects of T-algebras

Definition

Let $b : X \to B$ be a colax T-morphism[1]. We call **T-cartesianity defect of f at b** the canonical comparison map induced by the pullback below:



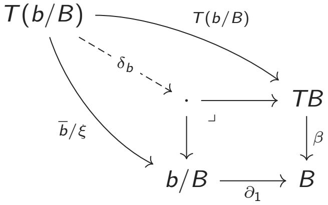
We say (B, β) is **T-cartesian at b** when its defect at b is invertible.

[1] Not really needed btw.

Cartesian objects of T-algebras

Definition

Let $b : X \to B$ be a colax T-morphism[1]. We call **T-cartesianity defect of f at b** the canonical comparison map induced by the pullback below:



We say (B, β) is **T-cartesian at b** when its defect at b is invertible.

Corollary

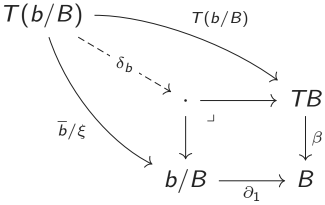
$B(b, -)$ is T-strong if and only if B is cartesian at b.

[1] Not really needed btw.

Cartesian objects of T-algebras

Definition

Let $b : {\color{red}1} \to B$ be a colax T-morphism[1]. We call **T-cartesianity defect of f at b** the canonical comparison map induced by the pullback below:



$$\delta_b : t(B(b, a_1), \ldots, B(b, a_n)) \to B(b, t(a_1, \ldots, a_n))$$

We say (B, β) is **T-cartesian at b** when its defect at b is invertible.

Corollary

$B(b, -)$ is T-strong if and only if B is cartesian at b.

[1] Not really needed btw.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- T = free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \overset{\cdot}{} \searrow \cdot} \underset{r}{\overset{l}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- T = free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \cdot \searrow \cdot} \underset{r}{\overset{l}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).
- T-algebra structure on Ω: $\mathbb{S}\mathbf{et} := \mathbb{S}\mathbf{pan}(\mathbf{Set}) \rightsquigarrow$ vindicating (Paré 2011).

Examples

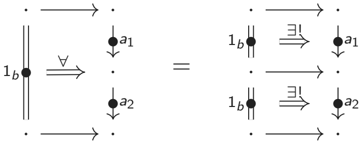
Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- T = free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \overset{\cdot}{} \searrow \cdot} \overset{l}{\underset{r}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).
- T-algebra structure on Ω: $\mathbb{S}\mathbf{et} := \mathbb{S}\mathbf{pan}(\mathbf{Set}) \rightsquigarrow$ vindicating (Paré 2011).
- $B(b, -)$ is strong double functor iff $B(1_b, a_1 \odot \cdots \odot a_n) \cong B(1_b, a_1) \times \cdots \times B(b, a_n)$.



Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- T = free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \cdot \searrow \cdot} \underset{r}{\overset{l}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).
- T-algebra structure on Ω: $\mathbb{S}\mathbf{et} := \mathbb{S}\mathbf{pan}(\mathbf{Set}) \rightsquigarrow$ vindicating (Paré 2011).
- $B(b, -)$ is strong double functor iff $B(1_b, a_1 \odot \cdots \odot a_n) \cong B(1_b, a_1) \underline{\times} \cdots \underline{\times} B(b, a_n)$.

Examples

Example (Symmetric monoidal categories)

- $T =$ free symmetric monoidal category 2-monad on $\mathbf{Cat}$, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- $T =$ free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \cdot \searrow \cdot} \overset{l}{\underset{r}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).
- T-algebra structure on Ω: $\mathbb{S}\mathbf{et} := \mathbb{S}\mathbf{pan}(\mathbf{Set}) \rightsquigarrow$ vindicating (Paré 2011).
- $B(b, -)$ is strong double functor iff $B(1_b, a_1 \odot \cdots \odot a_n) \cong B(1_b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **spanish double category**.

Examples

Example (Symmetric monoidal categories)

- T = free symmetric monoidal category 2-monad on **Cat**, with $\Omega = \mathbf{Set}$.
- T-algebra structure on Ω: $(\mathbf{Set}, 1, \times)$.
- $B(b, -)$ is strong monoidal iff $B(b, a_1 \otimes \cdots \otimes a_n) \cong B(b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **cartesian monoidal category**.

Example (Double categories)

- T = free double category 2-monad on $\mathbf{Cat}^{\cdot \rightrightarrows \cdot}$, with $\Omega = \mathbf{Set}^{\cdot \swarrow \cdot \searrow \cdot} \underset{r}{\overset{l}{\rightrightarrows}} \mathbf{Set}$ (see Mesiti 2024).
- T-algebra structure on Ω: $\mathbb{S}\mathbf{et} := \mathbb{S}\mathbf{pan}(\mathbf{Set}) \rightsquigarrow$ vindicating (Paré 2011).
- $B(b, -)$ is strong double functor iff $B(1_b, a_1 \odot \cdots \odot a_n) \cong B(1_b, a_1) \times \cdots \times B(b, a_n)$.

  All (global) objects cartesian: **spanish double category**.

Bonus fact: functors $p : E \to B$ = strict (tightly) discrete opfibrations $p : \ell E \to \ell B$

$\rightsquigarrow$ *Bénabou construction*: $\mathbf{Cat}/B \cong \mathbb{D}\mathbf{blCat}_{\mathrm{lx}}(\ell B, \mathbb{S}\mathbf{et})$

Thanks!

Enhanced 2-categories

Definition

An **enhanced 2-category**, or $\mathscr{F}$**-category**, is a 2-category $\mathcal{K} \equiv: \mathcal{K}_l$ whose 1-cells are called **loose** ($\rightsquigarrow$) and a wide and locally full subcategory $\mathcal{K}_t \hookrightarrow \mathcal{K}_l$. whose 1-cells are called **tight** ($\rightarrow$).

Definition

Enhanced 2-functors are 2-functors that preserve tightness, while **enhanced 2-natural transformations**, or **tight natural transformations**, are 2-natural transformations whose components are all tight.

Definition (Enhanced 2-monad)

An **enhanced 2-monad** is an $\mathscr{F}$-monad, thus a 2-monad (T, i, m) such that T preserves tightness and where i and m have tight components.

Enhanced 2-category of T-algebras and lax morphisms

Definition

The **enhanced 2-category of T-algebras and lax T-morphism** $\mathcal{A}\mathrm{lg}_l(T)$ for an enhanced 2-monad T on the enhanced 2-category $\mathcal{K}$ is the enhanced 2-category so comprised:

1. its objects are strict T-algebras whose structure map is tight in $\mathcal{K}$,
2. its loose maps are lax T-morphisms,

$$\begin{array}{ccc} TA & \overset{Tf}{\rightsquigarrow} & TB \\ {\scriptstyle\alpha}\downarrow & \swArrow & \downarrow{\scriptstyle\beta} \\ A & \underset{f}{\rightsquigarrow} & B \end{array}$$

3. its tight maps are strict T-morphisms whose underlying map is tight in $\mathcal{K}$,

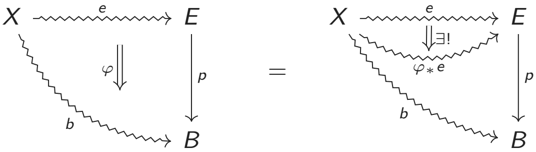
$$\begin{array}{ccc} TA & \xrightarrow{Tf} & TB \\ {\scriptstyle\alpha}\downarrow & & \downarrow{\scriptstyle\beta} \\ A & \xrightarrow[f]{} & B \end{array}$$

4. its 2-morphisms are T-2-morphisms.

TDOs

Definition ((Tight) discrete opfibration)

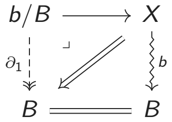
A **(tight) discrete opfibration** in a(n enhanced) 2-category $\mathcal{K}$ is a (tight) map $p : E \to B$ that admits unique (*opcartesian*) lifts:



Representable TDOs

Definition (Representable discrete opfibration)

A discrete opfibration is **representable** if it is equivalent to the projection out of a comma object dashed below:



We say $b/B \xrightarrow{\partial_1} B$ is **represented by the object** $b : X \to B$. When $X = 1$, we say it is **globally representable**. When $b = \mathrm{id}_B$, we get the **domain opfibration** associated to B.

TDOs classifier

This is an 'enhanced' and refined version of a definition from (Weber 2007) and (Mesiti 2024):

Definition (Enhanced 2-classifier)

An **enhanced 2-classifier** in an enhanced 2-category $\mathcal{K}$ with tight terminal object 1 and (left-tight) commas is a tight map

$$\tau : 1 \to \Omega$$

such that the functor $\tau/-$ induced by taking comma objects is amnestic, and fully faithful, and equipped with the structure of an adjoint equivalence:

$$\cong \circlearrowright \ \mathrm{tdos}(B) \underset{\tau/-}{\overset{\mathrm{d}(-)}{\rightleftarrows}} \mathcal{K}(B, \Omega) \ \circlearrowleft \cong$$

TDOs classifier

This is an 'enhanced' and refined version of a definition from (Weber 2007) and (Mesiti 2024):

Definition (Enhanced 2-classifier)

An **enhanced 2-classifier** in an enhanced 2-category $\mathcal{K}$ with tight terminal object 1 and (left-tight) commas is a tight map

$$\tau : 1 \to \Omega$$

such that the functor $\tau/-$ induced by taking comma objects is amnestic, and fully faithful, and equipped with the structure of an adjoint equivalence:

$$\cong \circlearrowright \ \mathrm{tdos}(B) \underset{\tau/-}{\overset{\mathrm{d}(-)}{\rightleftarrows}} \mathcal{K}(B, \Omega) \ \circlearrowleft \cong$$

Amnestic

$$\tau/f = \tau/f' \Rightarrow f = f',$$

Adjoint equivalence

$$\mathrm{d}(\tau/f) \cong f, \ \tau/\mathrm{d}p \cong p.$$

TDOs classifier

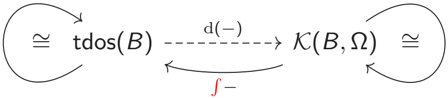
This is an 'enhanced' and refined version of a definition from (Weber 2007) and (Mesiti 2024):

Definition (Enhanced 2-classifier)

An **enhanced 2-classifier** in an enhanced 2-category $\mathcal{K}$ with tight terminal object 1 and (left-tight) commas is a tight map

$$\tau : 1 \to \Omega$$

such that the functor $\int -$ induced by taking comma objects is amnestic, and fully faithful, and equipped with the structure of an adjoint equivalence:



Amnestic

$$\int f = \int f' \Rightarrow f = f',$$

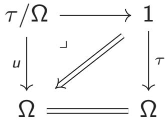
Adjoint equivalence

$$\mathrm{d}(\int f) \cong f, \ \int \mathrm{d}p \cong p.$$

A map $f : B \to \Omega$ is a **formal copresheaf** and $\int f$ its **category of elements**, while $\mathrm{d}p : B \to \Omega$ is the **copresheaf of fibers** of a tdos $p : E \to B$.

Generic discrete opfibration

The discrete opfibration classified by the identity or, equivalently, represented by τ, is called the **generic discrete opfibration**:



The setting of plumbuses

Definition (Plumbus)

A **plumbus** is an enhanced 2-category $\mathcal{K}$ admitting the following enhanced 2-limits:

1. all left-tight pullbacks of tight discrete opfibrations,
2. all left-tight commas,
3. a tight terminal object.

Let (T, i, m) be an enhanced 2-monad on $\mathcal{K}$.

Proposition

Let $p : (E, \eta) \to (B, \beta)$ be a strict T-morphism. Then if p is a tight discrete opfibration in $\mathcal{A}\mathrm{lg}_l(T)$, then so it is in $\mathcal{K}$.

Proposition

When $\mathcal{K}$ is a plumbus, the 2-category $\mathcal{A}\mathrm{lg}_l(T)$ is a plumbus too.

Opfibrantly cartesian 2-monads

An enhanced 2-monad ‘cartesian enough’ to lift a 2-classifier.

Definition (Opfibrantly cartesian enhanced 2-monad)

An enhanced 2-monad (T, i, m) is **opfibrantly cartesian** when

1. T preserves pullbacks of tight discrete opfibrations,
2. T preserves tight discrete opfibrations,
3. i and m are cartesian at all tight discrete opfibrations.

Opfibrantly cartesian 2-monads

An enhanced 2-monad 'cartesian enough' to lift a 2-classifier.

Definition (Opfibrantly cartesian enhanced 2-monad)

An enhanced 2-monad (T, i, m) is **opfibrantly cartesian** when

1. T preserves pullbacks of the **generic discrete opfibration** u,
2. Tu is a (tight) discrete opfibration,
3. i and m are cartesian at the **generic discrete opfibration** u.

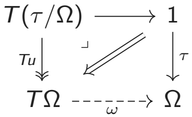
Opfibrantly cartesian 2-monads

An enhanced 2-monad 'cartesian enough' to lift a 2-classifier.

Definition (Opfibrantly cartesian enhanced 2-monad)

An enhanced 2-monad (T, i, m) is **opfibrantly cartesian** when

1. T preserves pullbacks of the **generic discrete opfibration** u,
2. Tu is a (tight) discrete opfibration,
3. i and m are cartesian at the **generic discrete opfibration** u.



Opfibrantly cartesian 2-monads

An enhanced 2-monad 'cartesian enough' to lift a 2-classifier.

Definition (Opfibrantly cartesian enhanced 2-monad)

An enhanced 2-monad (T, i, m) is **opfibrantly cartesian** when

1. T preserves pullbacks of the **generic discrete opfibration** u,
2. Tu is a (tight) discrete opfibration,
3. i and m are cartesian at the **generic discrete opfibration** u.

$$\begin{array}{ccc} T(\tau/\Omega) & \longrightarrow & 1 \\ {\scriptstyle Tu}\big\downarrow & \lrcorner \;\; \swarrow\!\!\!\swarrow & \big\downarrow{\scriptstyle \tau} \\ T\Omega & \underset{\omega}{\dashrightarrow} & \Omega \end{array}$$

Lemma

(Ω, ω) is a strict T-algebra, $\tau : 1 \to \Omega$ is a colax T-morphism, and (Ω, ω) is cartesian at τ.

Opfibrantly cartesian 2-monads

An enhanced 2-monad 'cartesian enough' to lift a 2-classifier.

Definition

An enhanced 2-monad (T, i, m) is **opfibrantly cartesian** if and only if:

1. T preserves pullbacks of the generic discrete opfibration,
2. Ω is equipped with a T-algebra ω which is cartesian at τ.

The theorem

Theorem (C., Myers)

Let T be an opfibrantly cartesian 2-monad on a plumbus $\mathcal{K}$ such that $\tau : 1 \to (\Omega, \omega)$ is a strict T-morphism. Then its enhanced 2-category of T-algebras and lax morphisms admits τ as enhanced 2-classifier.

Corollary

Let $p : E \to B$ be a tight discrete opfibration in $\mathcal{A}\mathrm{lg}_l(T)$. Its classifying map $\mathrm{d}p : B \to \Omega$ is strong if and only if p is T-cartesian. In particular, the representable copresheaf $B(b, -) : B \to \Omega$ associated to a representable discrete opfibration $b/B \xrightarrow{\partial_1} B$ is always a lax T-morphism, and it is strong precisely when B is T-cartesian at b.

References I

[1] L. Mesiti, *2-classifiers via dense generators and hofmann-streicher universe in stacks*, 2024. DOI: 10.48550/ARXIV.2401.16900. [Online]. Available: https://arxiv.org/abs/2401.16900.

[2] R. Paré, "Yoneda Theory for Double Categories", *Theory and Applications of Categories*, vol. 25, no. 17, pp. 436–489, Nov. 17, 2011. [Online]. Available: http://www.tac.mta.ca/tac/volumes/25/17/25-17.pdf.

[3] M. Weber, "Yoneda structures from 2-toposes", *Applied Categorical Structures*, vol. 15, no. 3, pp. 259–323, May 2007, ISSN: 1572-9095. DOI: 10.1007/s10485-007-9079-2. [Online]. Available: http://dx.doi.org/10.1007/s10485-007-9079-2.