Between Set and Bool: Categories of Aristotelian Diagrams

Alex De Klerck

PhD student on the ERC Starting Grant STARTDIALOG project

Joint work with Lorenz Demey, Leander Vignero

European Research Council

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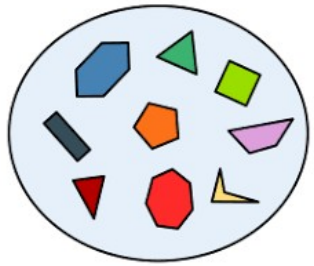
Set:



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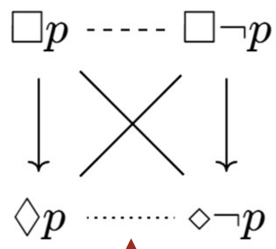
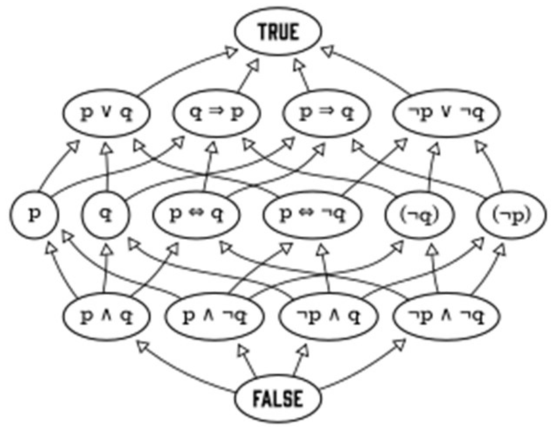
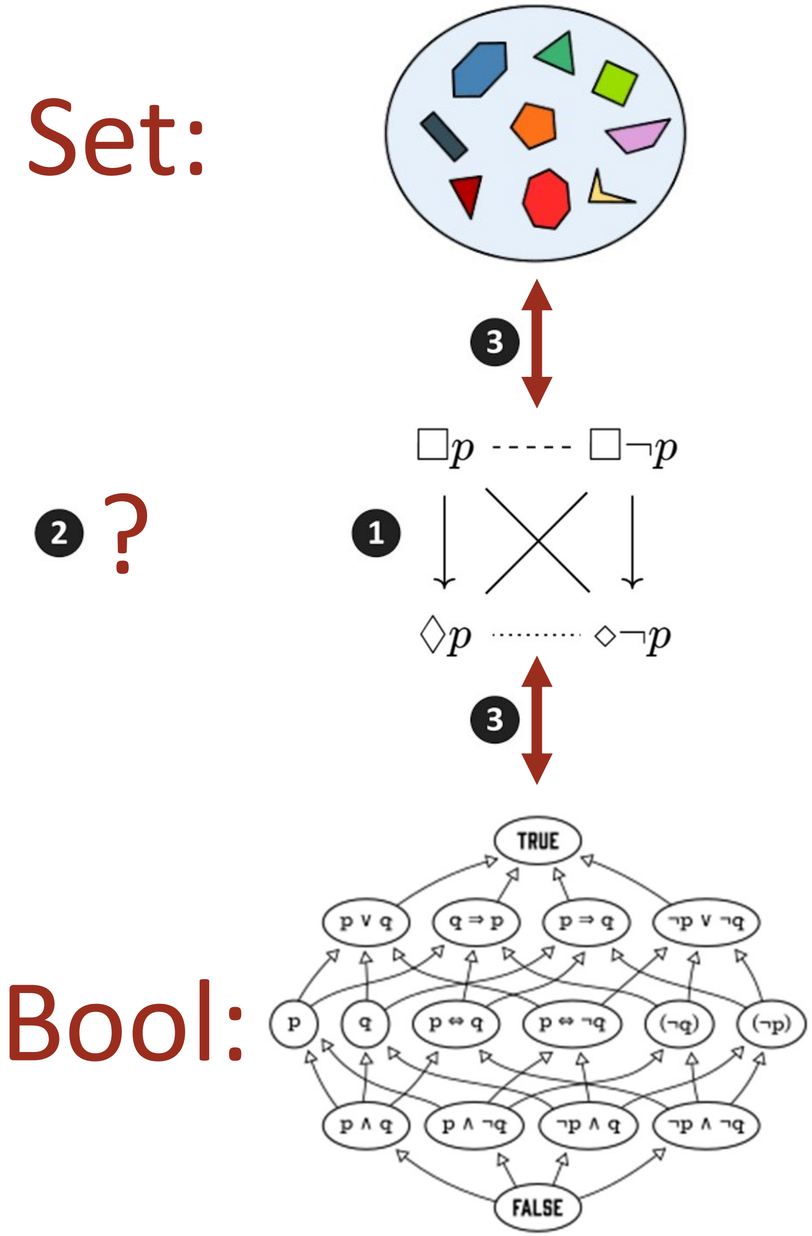


Bool:

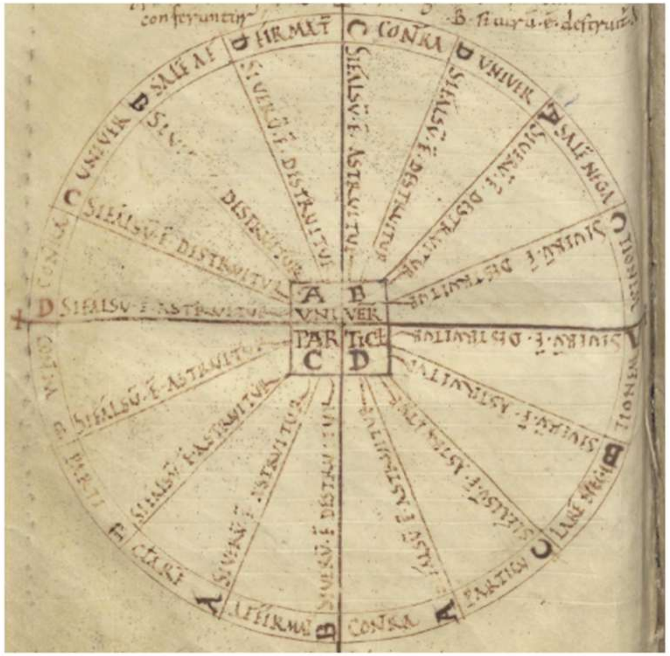


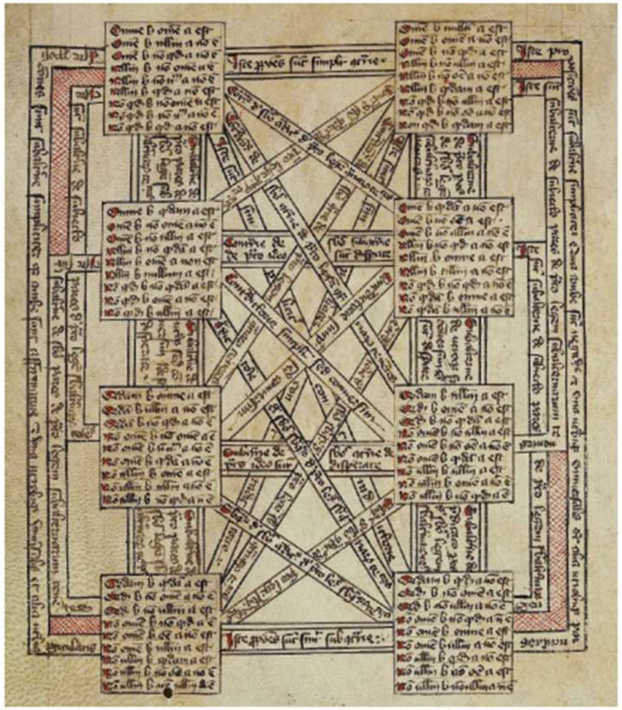
Overview

1. What are Aristotelian diagrams?
2. Categorification
3. Connections with Set and Bool

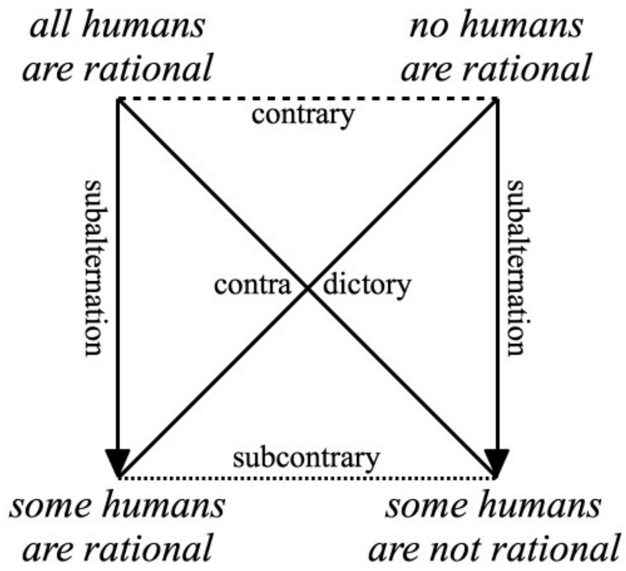


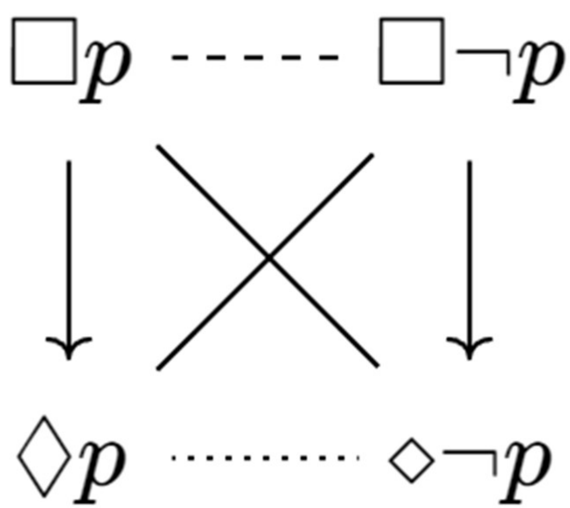
What are Aristotelian diagrams?

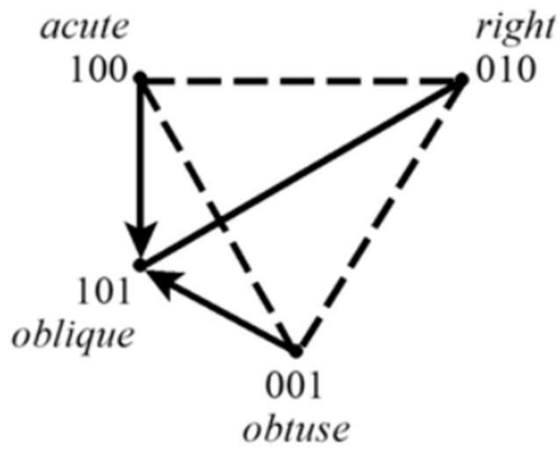




What are Aristotelian diagrams?







What are Aristotelian diagrams?

x **y**	Contradiction	**x,y** can both be true/false
x **y**	Contrariety	**x,y** can both be false, but not both true
x **y**	Subcontrariety	**x,y** can both be true, but not both false
x  **y**	Subalternation	**x** implies **y,** but not vice versa

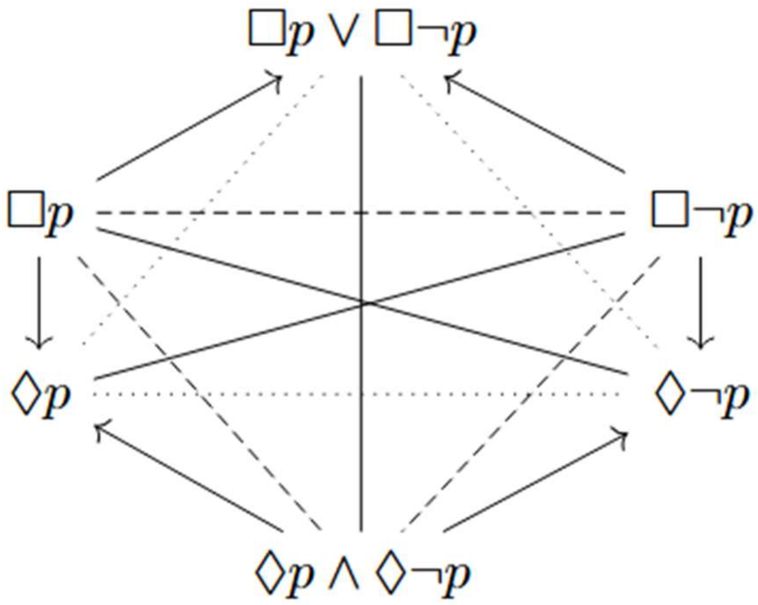
What are Aristotelian diagrams?

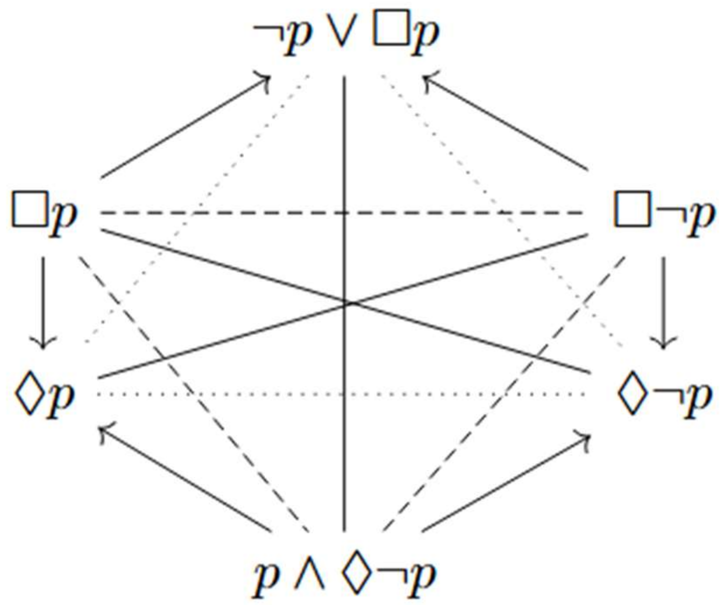
Definition. Given a Boolean algebra B, we say that $x, y \in B$ are:

- B-contradictory iff $x \wedge y = 0$ and $x \vee y = 1$,
- B-contrary iff $x \wedge y = 0$ and $x \vee y \neq 1$,
- B-subcontrary iff $x \wedge y \neq 0$ and $x \vee y = 1$,
- in B-subalternation iff $\neg x \vee y = 1$ and $x \vee \neg y \neq 1$.

Definition. An Aristotelian diagram D is a pair $(\mathcal{F}, B)$, where B is a Boolean algebra and $\mathcal{F}$ is a subset of B.

What are Aristotelian diagrams?





Similar Aristotelian diagrams, but different Boolean properties. Should they be considered isomorphic?

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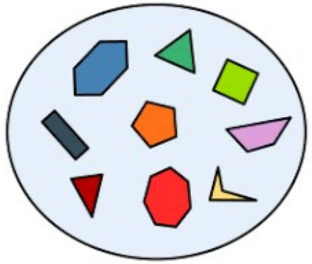

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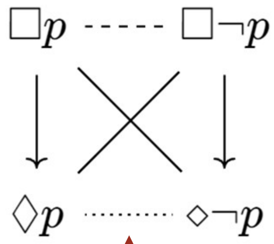

2 ?

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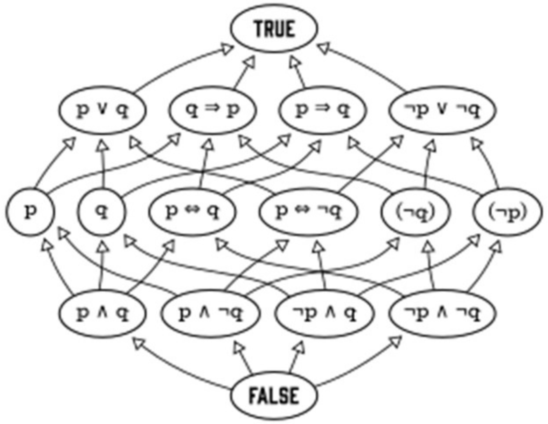
Bool:



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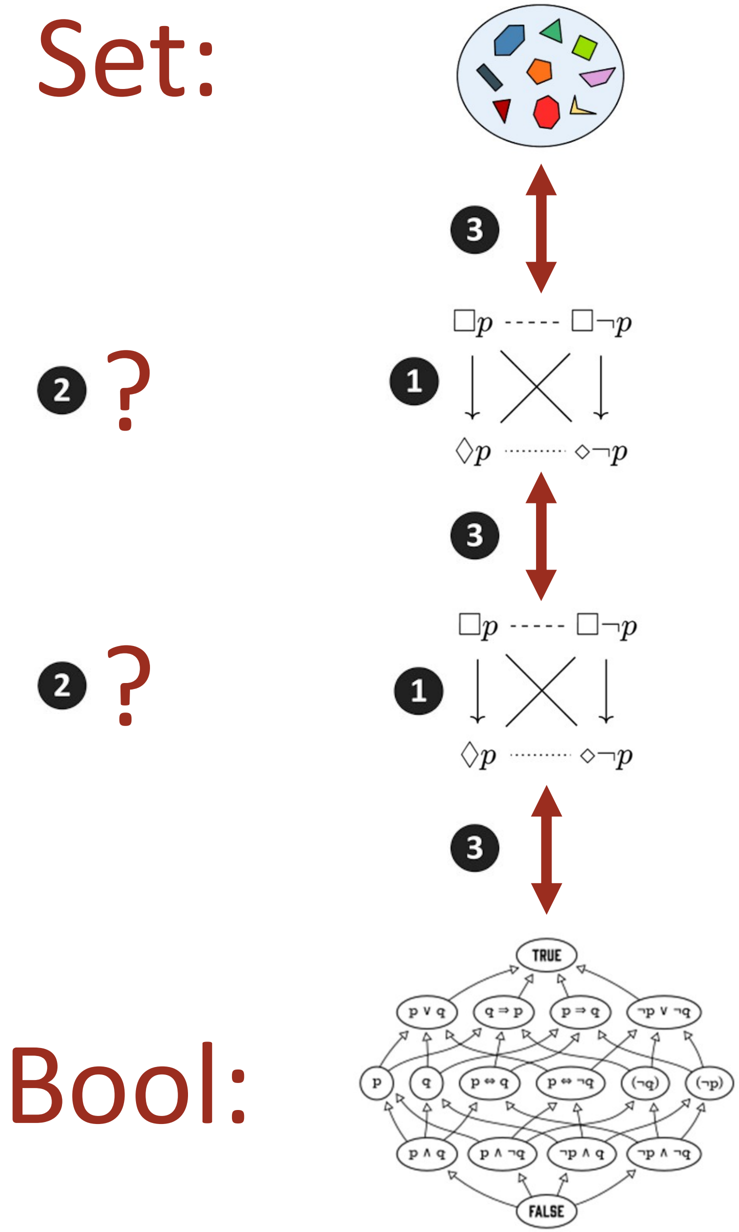

Categorification

	Relational structure level	Boolean structure level
Objects	Aristotelian diagrams	Aristotelian diagrams
Isomorphisms	Aristotelian isomorphisms	Boolean isomorphisms
Morphisms	?	?

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Morphisms Between Aristotelian Diagrams

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Volume 18, pages 49–83, (2024) Cite this article

Categorification

	Relational structure level	Boolean structure level
Objects	Aristotelian diagrams	Aristotelian diagrams
Isomorphisms	Aristotelian isomorphisms	Boolean isomorphisms
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Categorification

Definition. Let $D = (\mathcal{F}, B)$ and $D' = (\mathcal{F}', B')$ be Aristotelian diagrams. Then, a function $f : \mathcal{F} \to \mathcal{F}'$ is an increasing infomorphism from D to D' if and only if we have, for all $x, y \in \mathcal{F}$:

- $x \vee y = 1 \implies f(x) \vee f(y) = 1,$
- $\neg x \vee y = 1 \implies \neg f(x) \vee f(y) = 1,$
- $x \vee \neg y = 1 \implies f(x) \vee \neg f(y) = 1,$
- $\neg x \vee \neg y = 1 \implies \neg f(x) \vee \neg f(y) = 1.$

Categorification

	Relational structure level	Boolean structure level
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Categorification

	Relational structure level	Boolean structure level
Objects	Aristotelian diagrams	Aristotelian diagrams
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Morphisms	Increasing infomorphisms	Boolean morphisms

Definition. Let $(\mathcal{F}, B)$ and $(\mathcal{F}', B')$ be Aristotelian diagrams. A Boolean morphism $f : (\mathcal{F}, B) \to (\mathcal{F}', B')$ is a function $f : \mathcal{F} \to \mathcal{F}'$ such that there exists a Boolean algebra morphism $\varphi : Cl_B(\mathcal{F}) \to Cl_{B'}(\mathcal{F}')$ such that $f = \varphi|_{\mathcal{F}}$.

Categorification

	Relational structure level	Boolean structure level
Objects	Aristotelian diagrams	Aristotelian diagrams
Isomorphisms	Aristotelian isomorphisms	Boolean isomorphisms
Morphisms	Increasing infomorphisms	Boolean morphisms

$\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$ $\mathbb{D}_{\mathcal{B}}$

Categorification

	Relational structure level	Boolean structure level
Objects	Aristotelian diagrams	Aristotelian diagrams
Isomorphisms	Aristotelian isomorphisms	Boolean isomorphisms
Morphisms	Increasing infomorphisms	Boolean morphisms
Complete	✓	✓
Cocomplete	✓	✓

$\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$ $\mathbb{D}_{\mathcal{B}}$

Categorification

	Relational structure level	Boolean structure level
Terminal objects	$(\{*\}, B_*)$	$(\{*\}, B_*)$
Products	$(\mathcal{F} \times \mathcal{F}', B \times B')$	$(\mathcal{F} \times \mathcal{F}', B \times B')$
Equalizers	$(eq(f,g), B)$	$(eq(f,g), eq(\overline{f}, \overline{g}))$
Pullbacks	$(pb(f,g), B \times B')$	$(pb(f,g), pb(\overline{f}, \overline{g}))$
Initial Objects	$(\emptyset, B)$	$(\emptyset, B)$
Coproducts	$(\iota_B(\mathcal{F}) \cup \iota_{B'}(\mathcal{F}'), B + B')$	$(\iota_B(\mathcal{F}) \cup \iota_{B'}(\mathcal{F}'), B + B')$
Coequalizers	$(q_{\equiv_{coeq(f,g)}} \circ i_{\mathcal{F}}(\mathcal{F}), \frac{F(\mathcal{F})}{\equiv_{coeq(f,g)}})$	$(q_B(\mathcal{F}), coeq(\overline{f}, \overline{g}))$
Pushouts	$(q_{\equiv_{coeq(f_X,g_X)}} \circ i_X(X), \frac{F(X)}{\equiv_{coeq(f_X,g_X)}})$	$(q_{B+B'}(X), coeq(\overline{f_X}, \overline{g_X}))$

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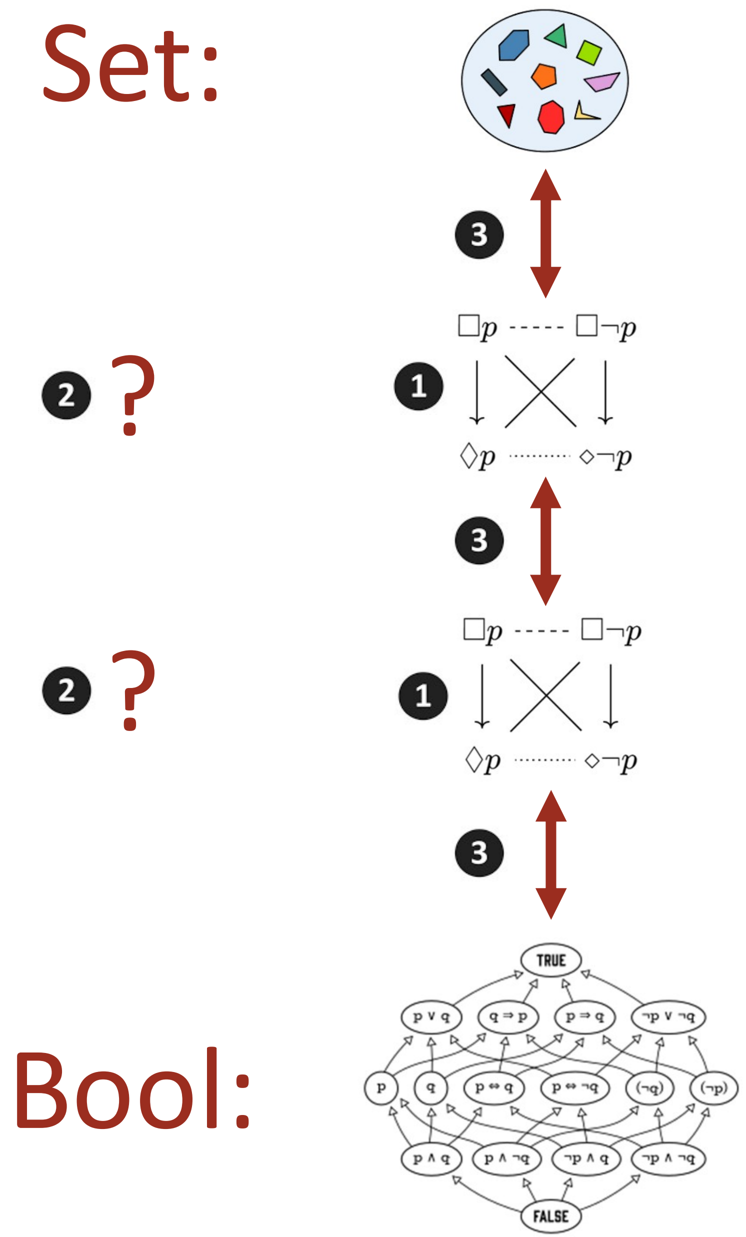

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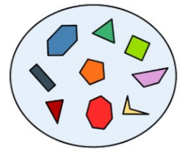

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2 $\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}}$

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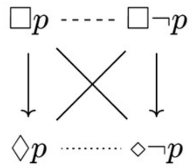

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2 $\mathbb{D}_{\mathcal{B}}$

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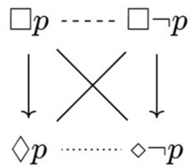

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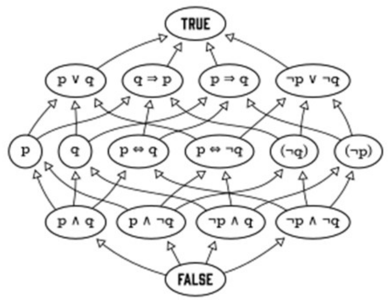
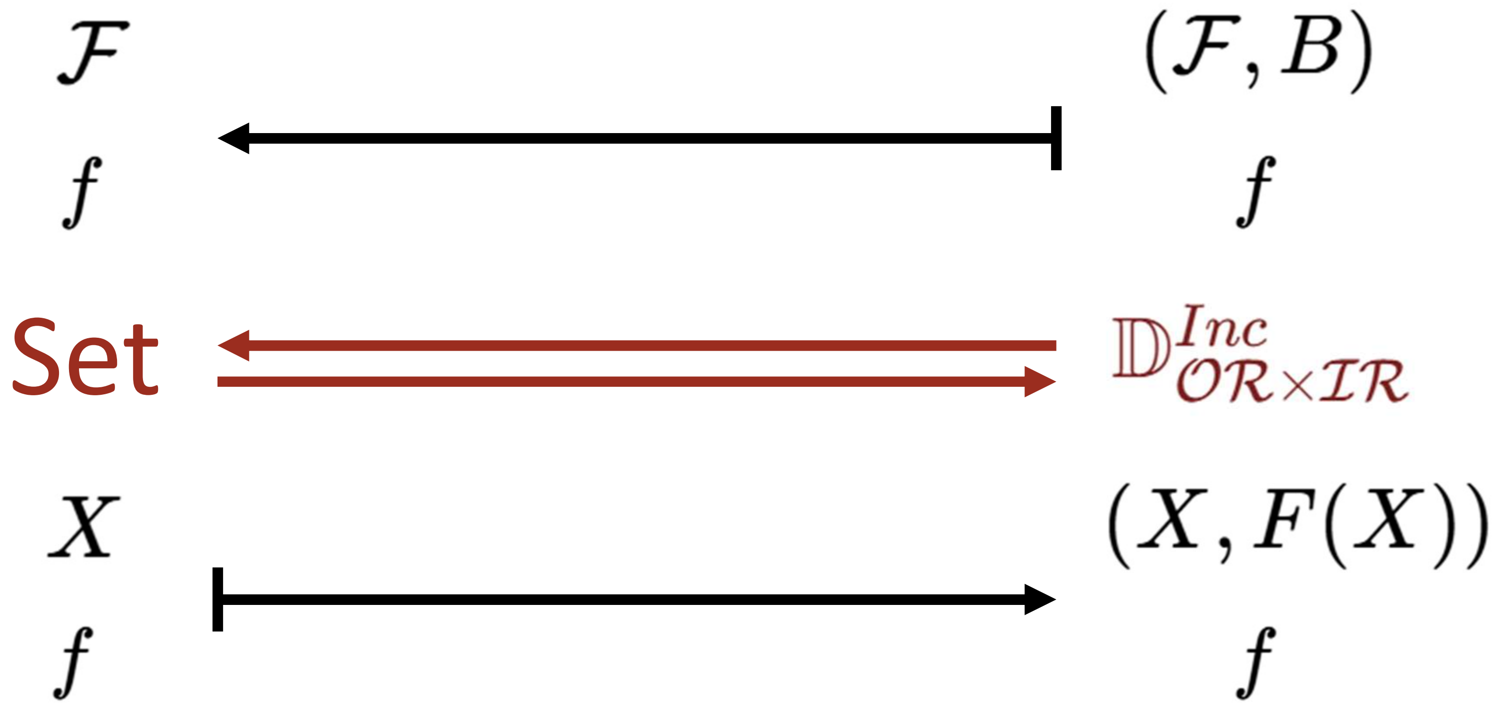
Bool:



Connections with Set and Bool



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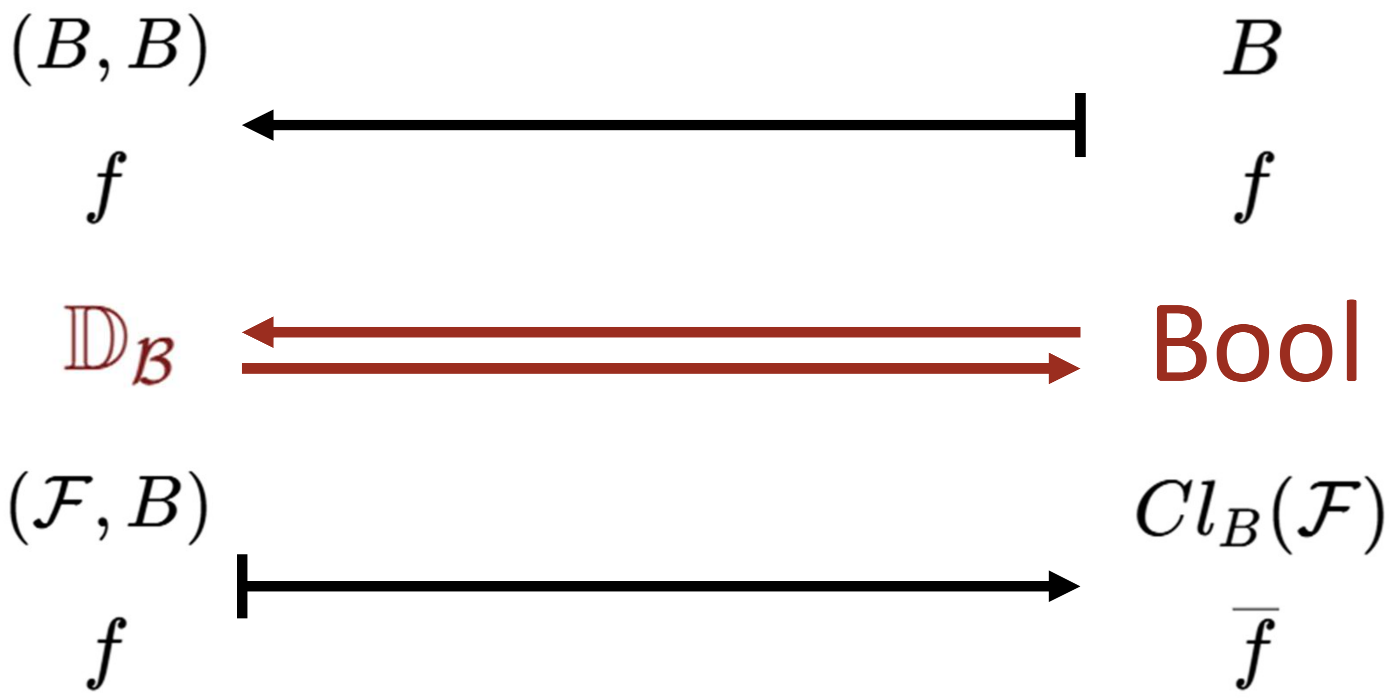


Connections with Set and Bool

$$(\mathcal{F}, B) \longleftarrow\!\!\shortmid (\mathcal{F}, B)$$

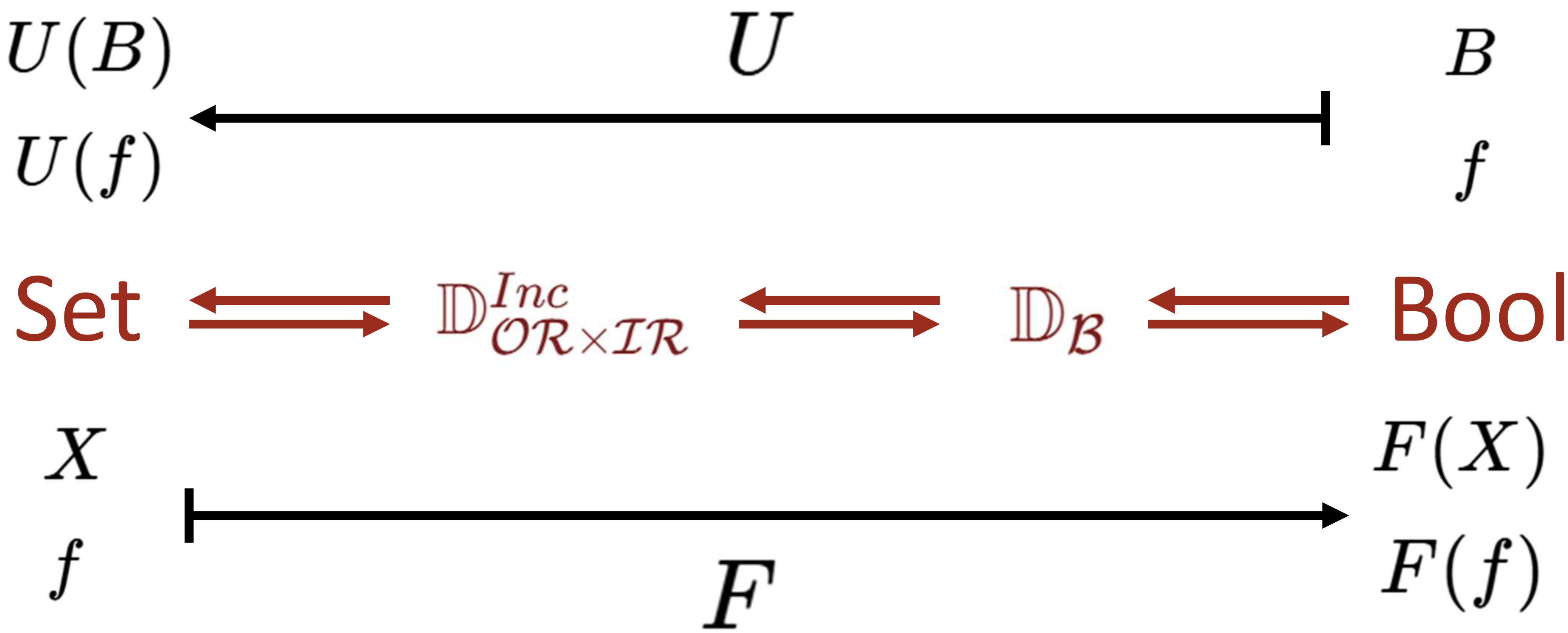
$$f \longleftarrow\!\!\shortmid f$$

$$\mathbb{D}^{Inc}_{\mathcal{OR}\times\mathcal{IR}} \rightleftarrows \mathbb{D}_{\mathcal{B}}$$

$$(\mathcal{F}, B) \longmapsto \left(q_{\equiv_{\mathcal{AD}_D}} \circ i_{\mathcal{F}}(\mathcal{F}), \frac{F(\mathcal{F})}{\equiv_{\mathcal{AD}_D}}\right)$$

$$f \longmapsto [i_{\mathcal{F}}(x)] \mapsto [i_{\mathcal{F}'}(f(x))]$$

Connections with Set and Bool



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