CATEGORY THEORY WITHIN A 2-CATEGORY

Λ

CAT_2

INTERNAL ENRICHMENT

$$\mathcal{C}^{op} \times \mathcal{C} \xrightarrow{\mathcal{C}(-,-)} \mathbb{S}$$

PRESHEAF OBJECTS

$$\mathcal{C} \hookrightarrow [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

AND CONVOLUTION PRODUCTS

$$\begin{tikzcd}
{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda \otimes_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[rr] \arrow[d] & & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \\
{[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \mathbb{S} \otimes_\Lambda \mathbb{S}]_\Lambda} \arrow[dr, "{[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \otimes_\mathbb{S}]_\Lambda}"'] & & \\
& {[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[uur, "\mathrm{LAN}^{\mathbb{S}}_{\otimes_{\mathcal{C}^{op}}}"'] &
\end{tikzcd}$$

CT 2025

SOPHIE D'ESPALUNGUE

CATEGORY THEORY

WITHIN

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CAT_2

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$$\mathcal{C}^{op} \times \mathcal{C} \xrightarrow{\mathcal{C}(-,-)} \mathbb{S}$$

PRESHEAF OBJECTS

$$\mathcal{C} \hookrightarrow [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

AND CONVOLUTION PRODUCTS

$$\begin{tikzcd}
{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda \otimes_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[rr] \arrow[d] & & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \\
{[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \mathbb{S} \otimes_\Lambda \mathbb{S}]_\Lambda} \arrow[r, "{[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \otimes_{\mathbb{S}}]_\Lambda}"'] & {[\mathcal{C}^{op} \otimes_\Lambda \mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[ur, "\mathrm{LAN}^{\mathbb{S}}_{\otimes_{\mathcal{C}^{op}}}"'] &
\end{tikzcd}$$

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CAT_2

Λ

Λ

CAT

CAT_2

Λ

$$\Lambda^{op} \times \Lambda \xrightarrow{\Lambda(-,-)} \mathrm{CAT}$$

CAT_2

Λ

$\Lambda^{op} \times \Lambda \xrightarrow{\Lambda(-,-)} \mathrm{CAT}$

CAT_2

$$\Lambda^{op} \times \Lambda \xrightarrow{\Lambda(-,-)} \mathrm{CAT}$$

CAT_2

$$* \xrightarrow{c} \Lambda$$

$\mathcal{C}$

$$
\begin{tikzcd}
\Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] & \mathcal{CAT}
\end{tikzcd}
$$

$\mathcal{CAT}_2$

$$
\begin{tikzcd}
* \arrow[r, bend left, "\mathcal{C}"] \arrow[r, bend right, "\mathcal{D}"'] \arrow[r, phantom, "F \Downarrow \overset{\Rrightarrow}{\eta} \Downarrow G"] & \Lambda
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D}
\end{tikzcd}
$$

$$
\begin{tikzcd}
* \arrow[rr, bend left=50, "*_\Lambda"] \arrow[rr, "\mathcal{C}"] \arrow[rr, bend right=50, "\mathcal{D}"'] \arrow[rr, phantom, bend right=25, "F \Downarrow \overset{\Rrightarrow}{\eta} \Downarrow G"] & & \Lambda
\end{tikzcd}
$$

$$
\begin{tikzcd}
\Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] & \mathrm{CAT}
\end{tikzcd}
$$

CAT_2

$*_\Lambda$

$$
\begin{tikzcd}
\mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D}
\end{tikzcd}
$$

$$\begin{tikzcd} * \arrow[rr, bend left=50, "*_\Lambda"] \arrow[rr, "\mathcal{C}"] \arrow[rr, bend right=50, "\mathcal{D}"'] & {\scriptstyle F \Downarrow \overset{\Rrightarrow}{\eta} \Downarrow G} & \Lambda \end{tikzcd} \xrightarrow{\cong} * \times \Lambda \xrightarrow{*_\Lambda \times \Lambda} \Lambda^{op} \times \Lambda \xrightarrow{\Lambda(-,-)} \mathrm{CAT}$$

CAT_2

$*_\Lambda$

$$\begin{tikzcd} \mathcal{C} \arrow[r, bend left=40, "F"] \arrow[r, bend right=40, "G"'] \arrow[r, phantom, "\eta\Downarrow"] & \mathcal{D} \end{tikzcd}$$

$$
\begin{tikzcd}
* \arrow[rr, bend left=50, "*_\Lambda"] \arrow[rr, "\mathcal{C}"] \arrow[rr, bend right=50, "\mathcal{D}"'] & \eta & \Lambda \arrow[r, "\cong"] \arrow[rrr, bend right=15, "\Lambda(*_\Lambda, -)"'] & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] & \mathcal{C}\mathrm{AT}
\end{tikzcd}
$$

F ⇓ ⇛ ⇓ G

$\mathcal{C}\mathrm{AT}_2$

$*_\Lambda$

$$
\begin{tikzcd}
\mathcal{C} \arrow[r, bend left=40, "F"] \arrow[r, bend right=40, "G"'] \arrow[r, phantom, "\eta\Downarrow"] & \mathcal{D}
\end{tikzcd}
$$

$\mathcal{C}^{*} := \Lambda(*_\Lambda, \mathcal{C})$

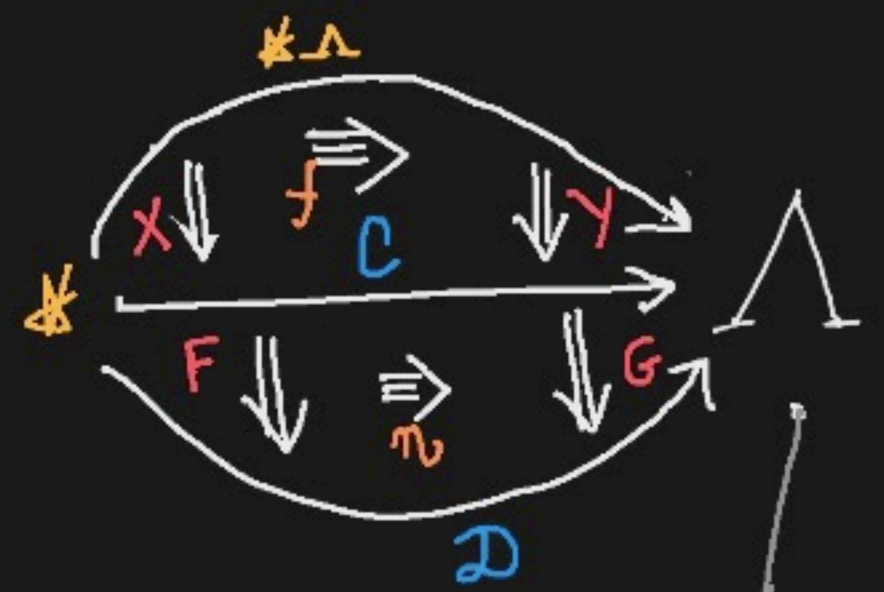
CAT_2

$$
\begin{tikzcd}[column sep=large]
* \arrow[r, bend left=60, "*_\Lambda"] \arrow[r, "\mathcal{C}" description] \arrow[r, bend right=60, "\mathcal{D}"'] \arrow[r, phantom, shift left=4, "X \Downarrow \; \overset{f}{\Rrightarrow} \; \Downarrow Y"] \arrow[r, phantom, shift right=4, "F \Downarrow \; \overset{\eta}{\Rrightarrow} \; \Downarrow G"] & \Lambda
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] & \\
\Lambda \arrow[ur, "\cong"] \arrow[rrr, "\Lambda(*_\Lambda, -)"'] & & & \mathrm{CAT}
\end{tikzcd}
$$

$$
\begin{tikzcd}
*_\Lambda \arrow[r, bend left=40, "X"] \arrow[r, bend right=40, "Y"'] \arrow[r, phantom, "f \Downarrow"] & \mathcal{C} \arrow[r, bend left=40, "F"] \arrow[r, bend right=40, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D}
\end{tikzcd}
$$

$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$



$$
\begin{tikzcd}
 & \star \times \Lambda \arrow[r, "\star_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] & \\
\Lambda \arrow[ur, "\simeq"] \arrow[rrr, "\Lambda(\star_\Lambda, -)"'] & & & \mathrm{CAT}
\end{tikzcd}
$$

CAT_2

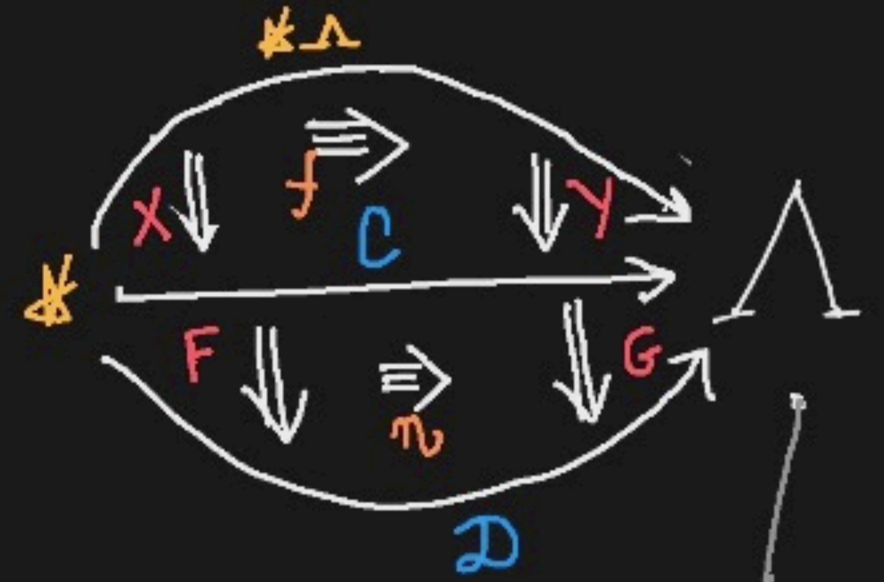
$$
\begin{tikzcd}
\star_\Lambda \arrow[r, bend left, "X"] \arrow[r, bend right, "Y"'] \arrow[r, phantom, "f \Downarrow"] & \mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D}
\end{tikzcd}
$$

INSIDE OUT

$\star \longrightarrow \odot$

$\mathcal{C}^{\star} := \Lambda(\star_\Lambda, \mathcal{C})$

CAT_2



$$\begin{tikzcd}
 & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] & \\
\Lambda \arrow[ur, "\simeq"] \arrow[rrr, "\Lambda(*_\Lambda, -)"'] & & & \mathrm{CAT}
\end{tikzcd}$$

$$\begin{tikzcd}
*_\Lambda \arrow[r, bend left, "X"] \arrow[r, bend right, "Y"'] \arrow[r, phantom, "f\Downarrow"] & \mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\eta\Downarrow"] & \mathcal{D}
\end{tikzcd}$$

$\boxed{X \xrightarrow{f} Y}$

$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$

$\boxed{X \xrightarrow{f} Y}$

INSIDE OUT

$* \longrightarrow \odot$

CAT_2

$$\begin{tikzcd}
& * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] & \\
\Lambda \arrow[ur, "\cong"] \arrow[rrr, "\Lambda(*_\Lambda, -)"'] & & & \mathrm{CAT}
\end{tikzcd}$$

$$\begin{tikzcd}
* \arrow[r, bend left=50, "*_\Lambda"] \arrow[r, "\mathcal{C}" description] \arrow[r, bend right=50, "\mathcal{D}"'] & \Lambda
\end{tikzcd}$$

$X \Downarrow \quad \overset{f}{\Rrightarrow} \quad \Downarrow Y$

$F \Downarrow \quad \overset{\eta}{\Rrightarrow} \quad \Downarrow G$

$$\begin{tikzcd}
*_\Lambda \arrow[r, bend left, "X"] \arrow[r, bend right, "Y"'] \arrow[r, phantom, "f \Downarrow"] & \mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D}
\end{tikzcd}$$

$$\begin{tikzcd}
Fx \arrow[r, "\eta_x"] \arrow[d, "Ff"'] \arrow[dr, dashed, "\eta_f" description] \arrow[r, phantom, shift right=4, "="] & Gx \arrow[d, "Gf"] \\
Fy \arrow[r, "\eta_y"'] & Gy
\end{tikzcd}$$

$X \xrightarrow{f} Y$

INSIDE OUT

$* \longrightarrow \odot$

$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$

$X \xrightarrow{f} Y$

$$\begin{tikzcd}[column sep=huge]
* \arrow[r, bend left=70, "*_\Lambda"{name=A}] \arrow[r, "\mathcal{C}"{name=B, description}] \arrow[r, bend right=70, "\mathcal{D}"'{name=C}] & \Lambda
\arrow[from=A, to=B, phantom, "X \Downarrow \overset{f}{\Rrightarrow} \Downarrow Y"]
\arrow[from=B, to=C, phantom, "F \Downarrow \overset{\eta}{\Rrightarrow} \Downarrow G"]
\end{tikzcd}$$

$$\begin{tikzcd}
& * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] & \\
\Lambda \arrow[ur, "\simeq"] \arrow[rrr, "\Lambda(*_\Lambda, -)"'] & & & \mathrm{CAT}
\end{tikzcd}$$

CAT_2

$$\begin{tikzcd}
*_\Lambda \arrow[r, bend left, "X"{name=X}] \arrow[r, bend right, "Y"'{name=Y}] & \mathcal{C} \arrow[r, bend left, "F"{name=F}] \arrow[r, bend right, "G"'{name=G}] & \mathcal{D}
\arrow[from=X, to=Y, Rightarrow, "f"']
\arrow[from=F, to=G, Rightarrow, "\eta"']
\end{tikzcd}$$

$X \xrightarrow{f} Y$

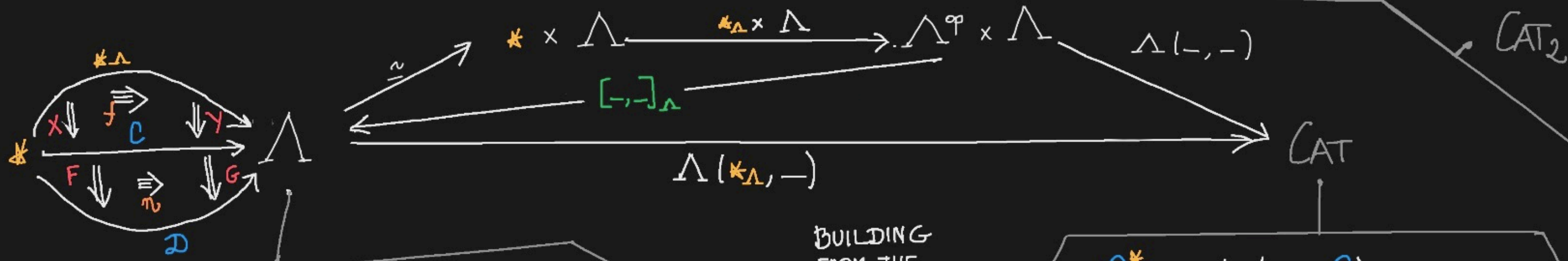
$$\begin{tikzcd}
Fx \arrow[r, "\eta_x"] \arrow[d, "Ff"'] \arrow[dr, dashed, "\eta * f" description] & Gx \arrow[d, "Gf"] \\
Fy \arrow[r, "\eta_y"'] & Gy
\end{tikzcd}$$

BUILDING
FROM THE
POINT $*$

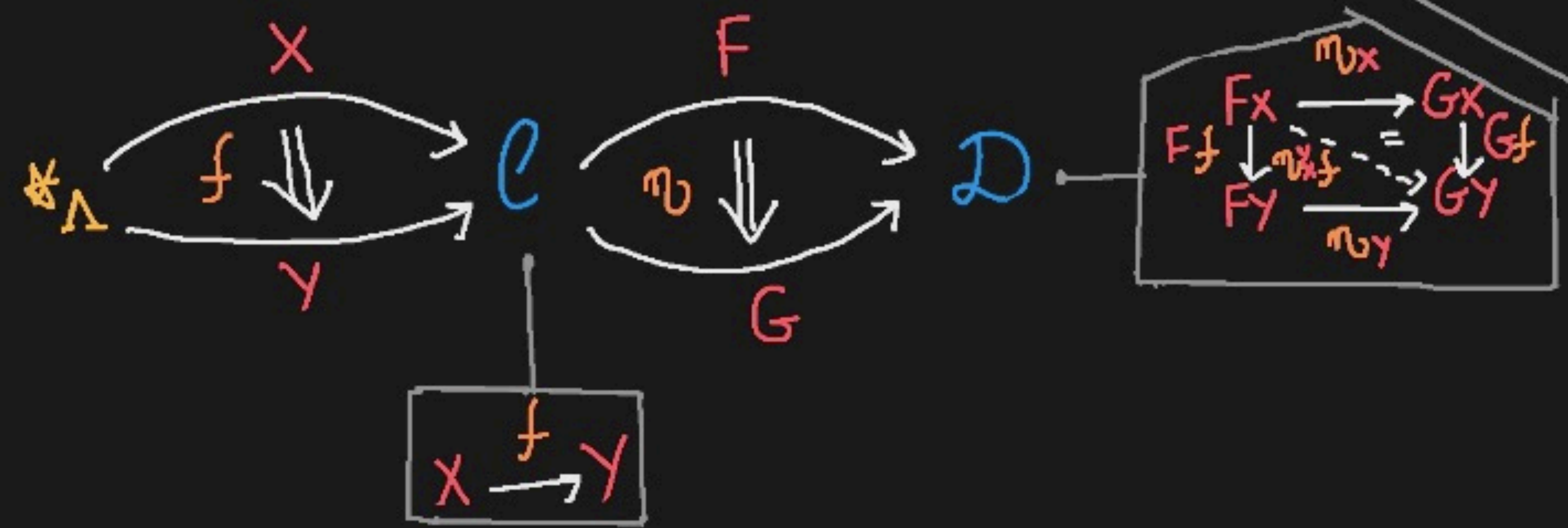
INSIDE OUT
$* \longrightarrow \odot$

$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$

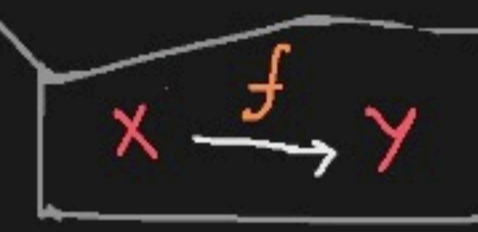
$X \xrightarrow{f} Y$



BUILDING FROM THE POINT $*$

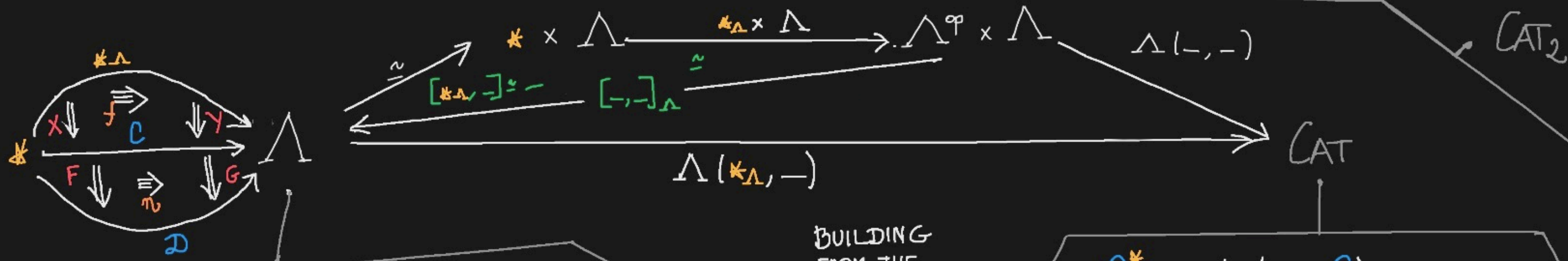


$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$

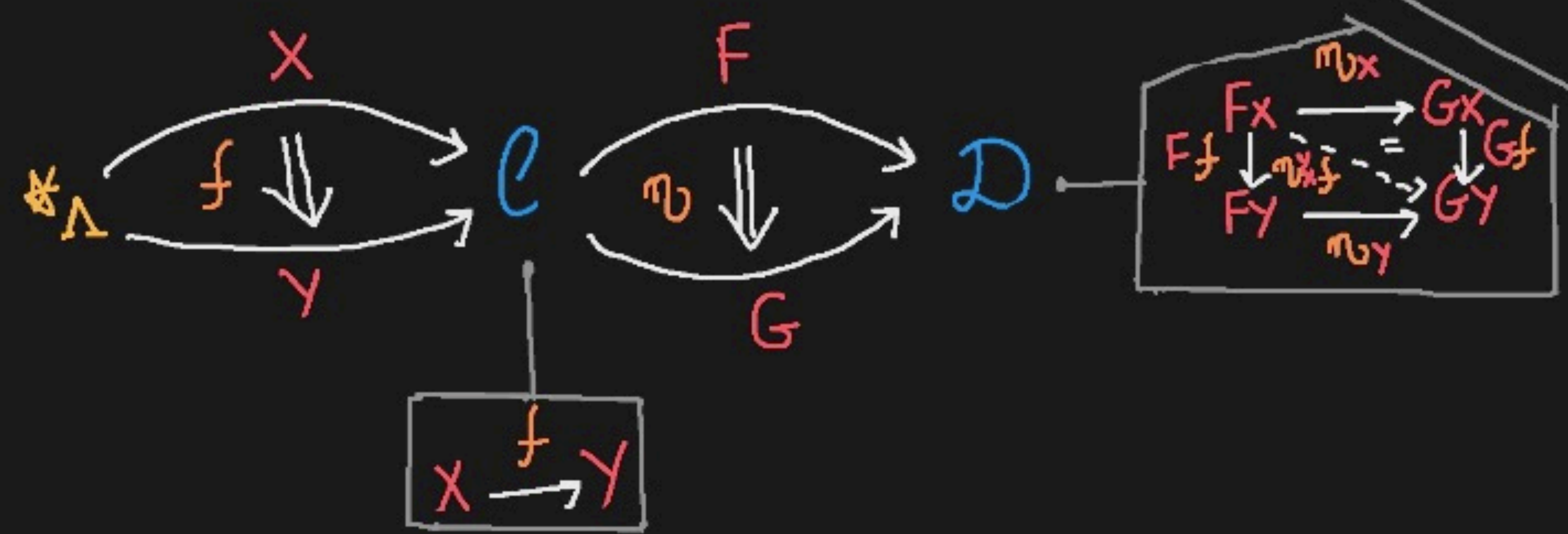


INSIDE OUT

$* \longrightarrow \odot$



BUILDING FROM THE POINT $*$

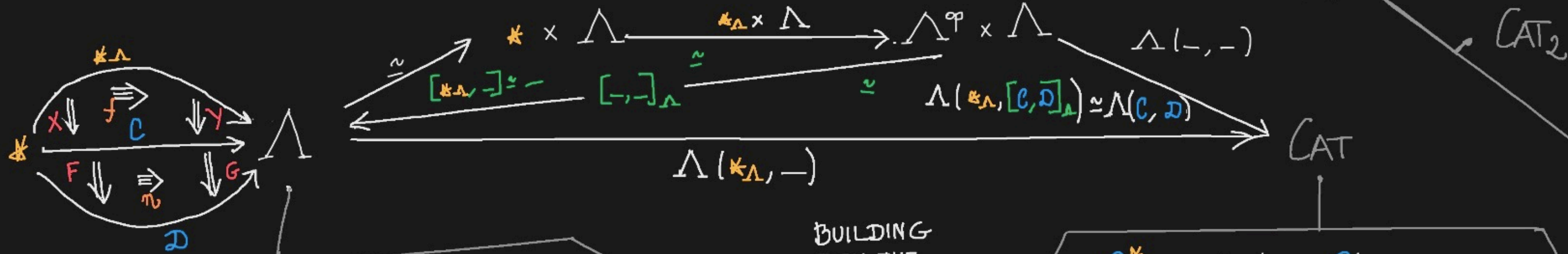


INSIDE OUT

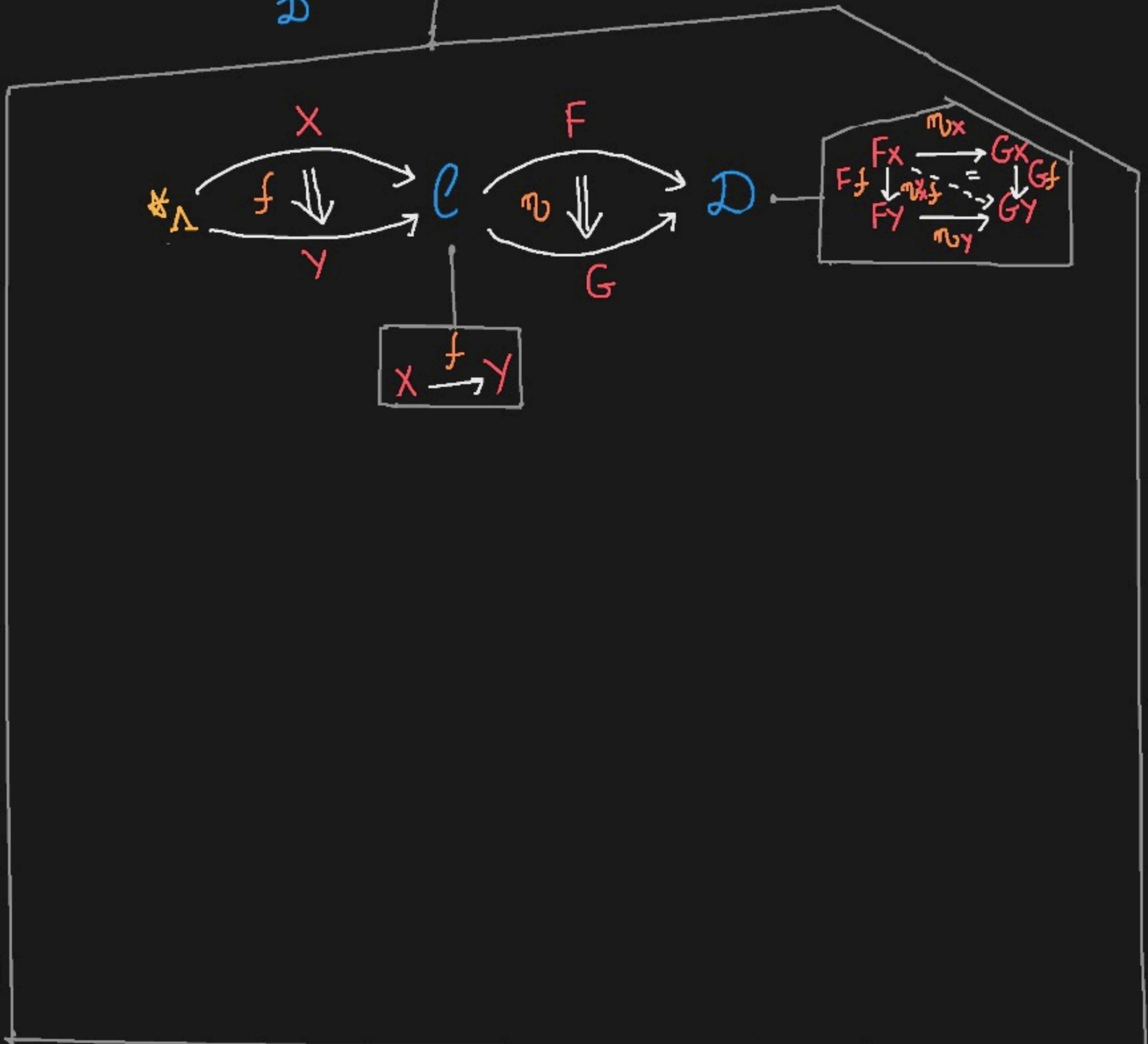
$* \longrightarrow \odot$

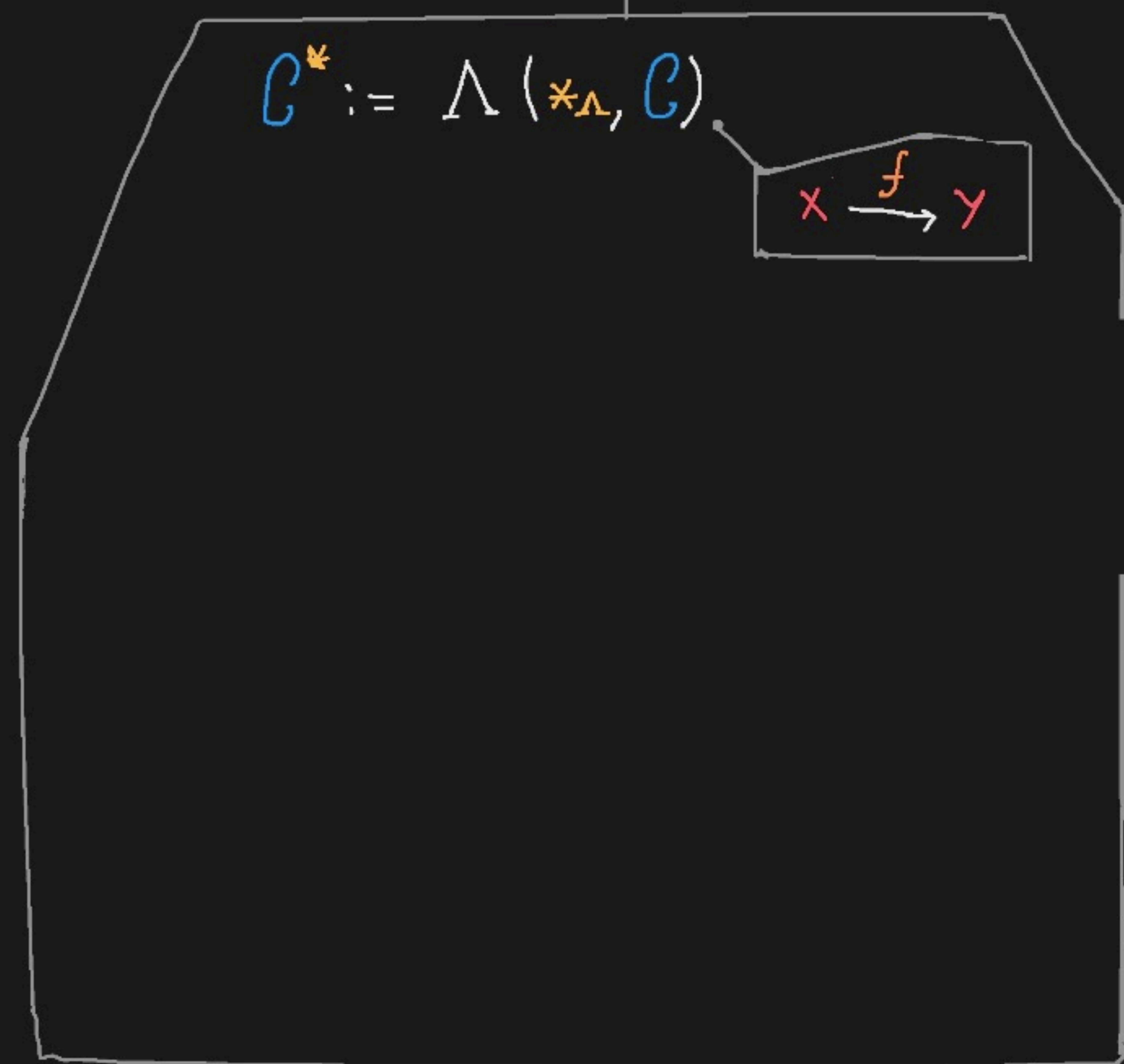
$C^* := \Lambda(*_\Lambda, C)$

$X \xrightarrow{f} Y$



BUILDING FROM THE POINT $*$





INSIDE OUT

$* \longrightarrow \odot$

$$\begin{tikzcd}
& {*} \times \Lambda \arrow[rr, "*_\Lambda \times \Lambda"] & & \Lambda^{op} \times \Lambda \arrow[ddr, "\Lambda(-,-)"] \arrow[dll, "{[-,-]_\Lambda}"', "\simeq"] & \\
& {[*_\Lambda, -] \simeq -} & & & \\
\Lambda \arrow[uur, "\simeq"] \arrow[rrrr, "{\Lambda(*_\Lambda, -)}"'] & & & & \mathrm{CAT}
\end{tikzcd}$$

$$\Lambda(*_\Lambda, [\mathcal{C}, \mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C}, \mathcal{D})$$

CAT_2

$$\begin{tikzcd}
{*} \arrow[rr, bend left=60, "*_\Lambda"] \arrow[rr, "\mathcal{C}"] \arrow[rr, bend right=60, "\mathcal{D}"'] & & \Lambda
\end{tikzcd}$$

$X \Downarrow \; \overset{f}{\Rrightarrow} \; \Downarrow Y \qquad F \Downarrow \; \overset{\eta}{\Rrightarrow} \; \Downarrow G$

$$\begin{tikzcd}
*_\Lambda \arrow[r, bend left=40, "X"] \arrow[r, bend right=40, "Y"'] \arrow[r, phantom, "f \Downarrow"] \arrow[d, bend right=20, "F"'] \arrow[d, bend left=20, "G"] \arrow[d, phantom, "\overset{\eta}{\Rightarrow}"] & \mathcal{C} \arrow[r, bend left=40, "F"] \arrow[r, bend right=40, "G"'] \arrow[r, phantom, "\eta \Downarrow"] & \mathcal{D} \\
{[\mathcal{C}, \mathcal{D}]_\Lambda} & &
\end{tikzcd}$$

$$X \xrightarrow{f} Y$$

$$\begin{tikzcd}
Fx \arrow[r, "\eta_x"] \arrow[d, "Ff"'] \arrow[dr, dashed, "\eta * f"] & Gx \arrow[d, "Gf"] \\
Fy \arrow[r, "\eta_y"'] & Gy
\end{tikzcd}$$

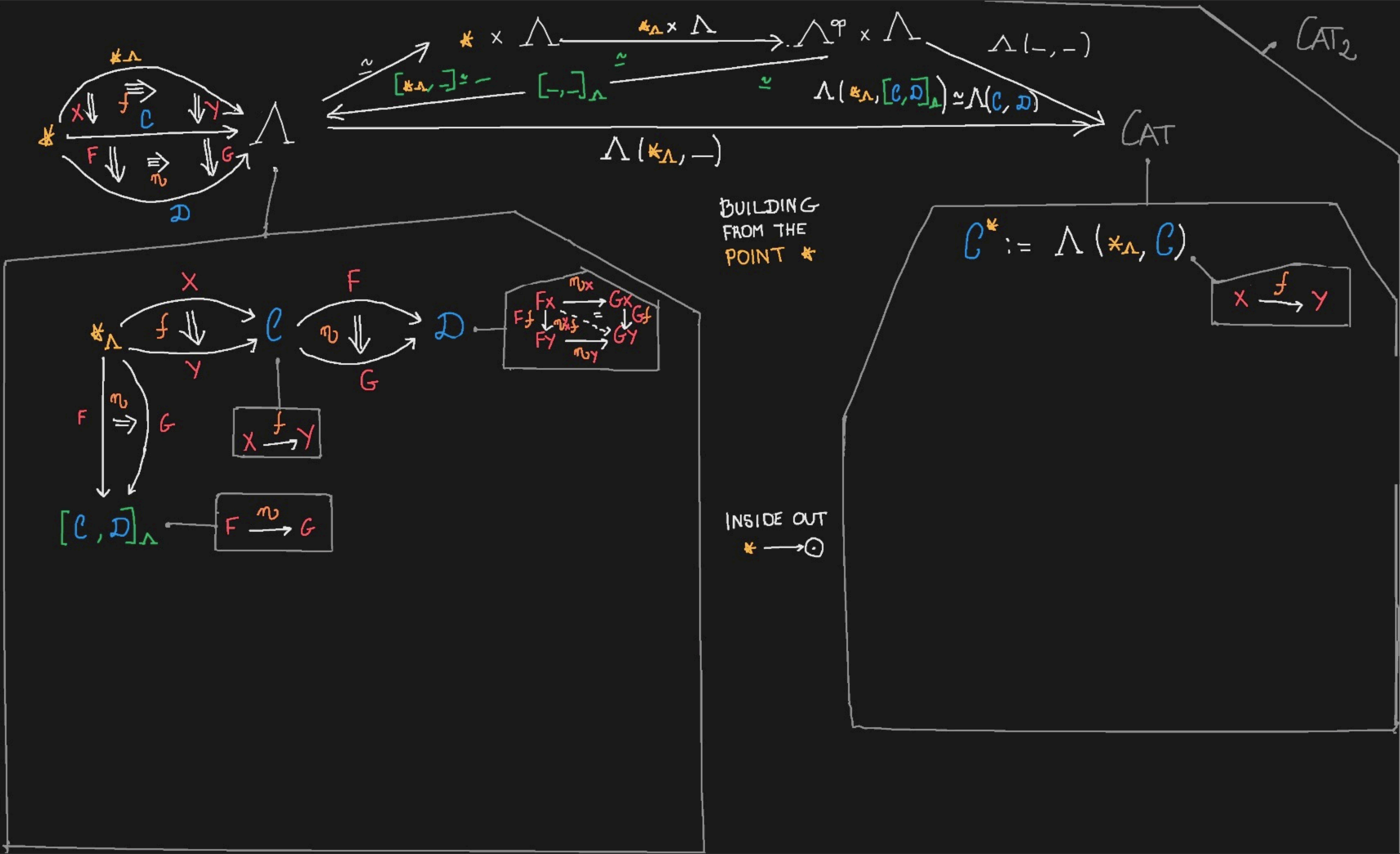
BUILDING FROM THE POINT $*$

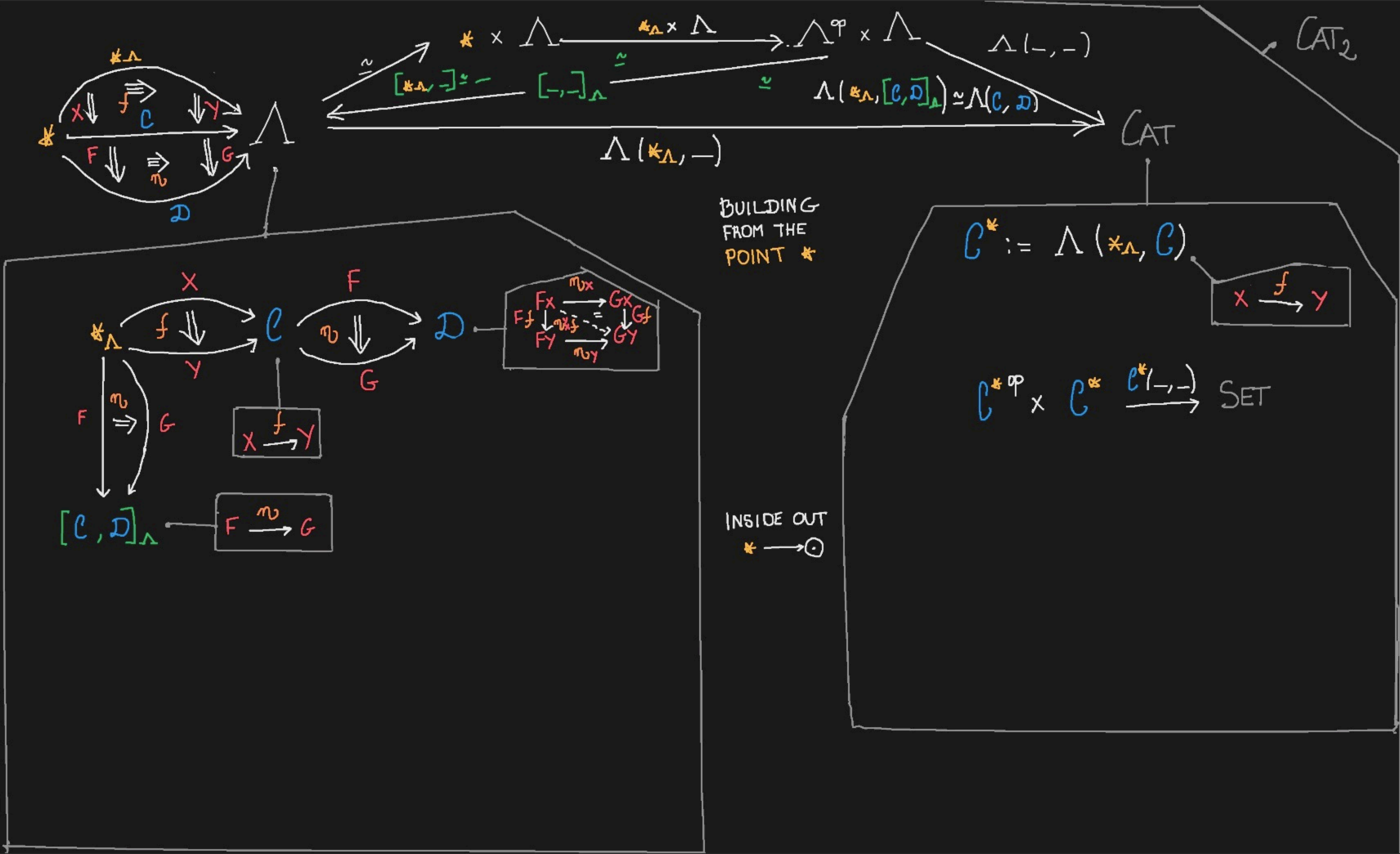
INSIDE OUT

$* \longrightarrow \odot$

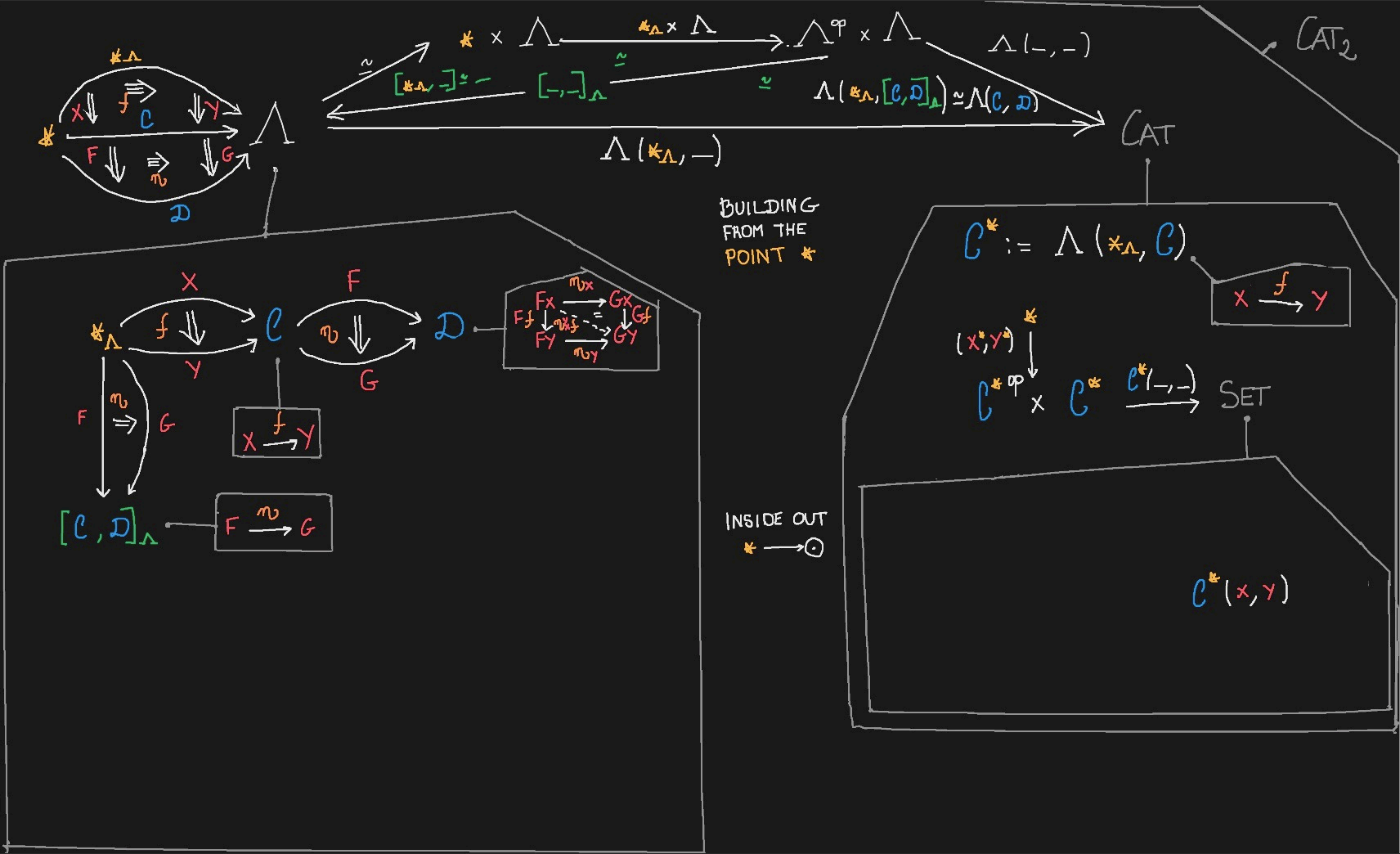
$$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$$

$$X \xrightarrow{f} Y$$

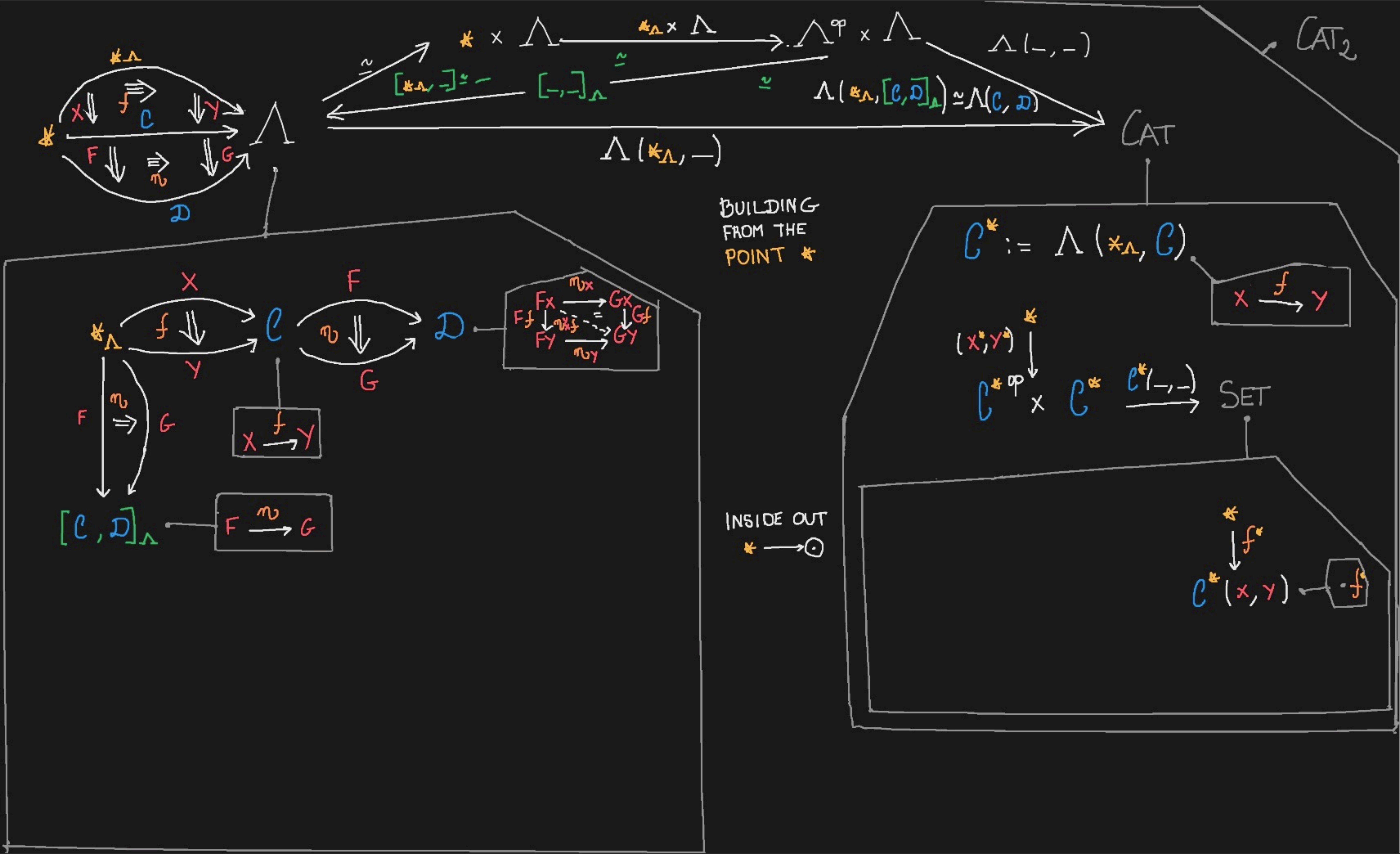

CAT₂
CAT
Λ(−, −)
Λ(*_Λ, −)
Λ(*_Λ, [C, D]_Λ) ≃ Λ(C, D)
*_Λ × Λ
Λ^op × Λ
[*_Λ, −] ≃ −
[−, −]_Λ
BUILDING FROM THE POINT *
INSIDE OUT
* ⟶ ⊙
C* := Λ(*_Λ, C)
X —f→ Y
F —η→ G
[C, D]_Λ
ηx
ηy
Fx
Gx
Fy
Gy
Ff
Gf

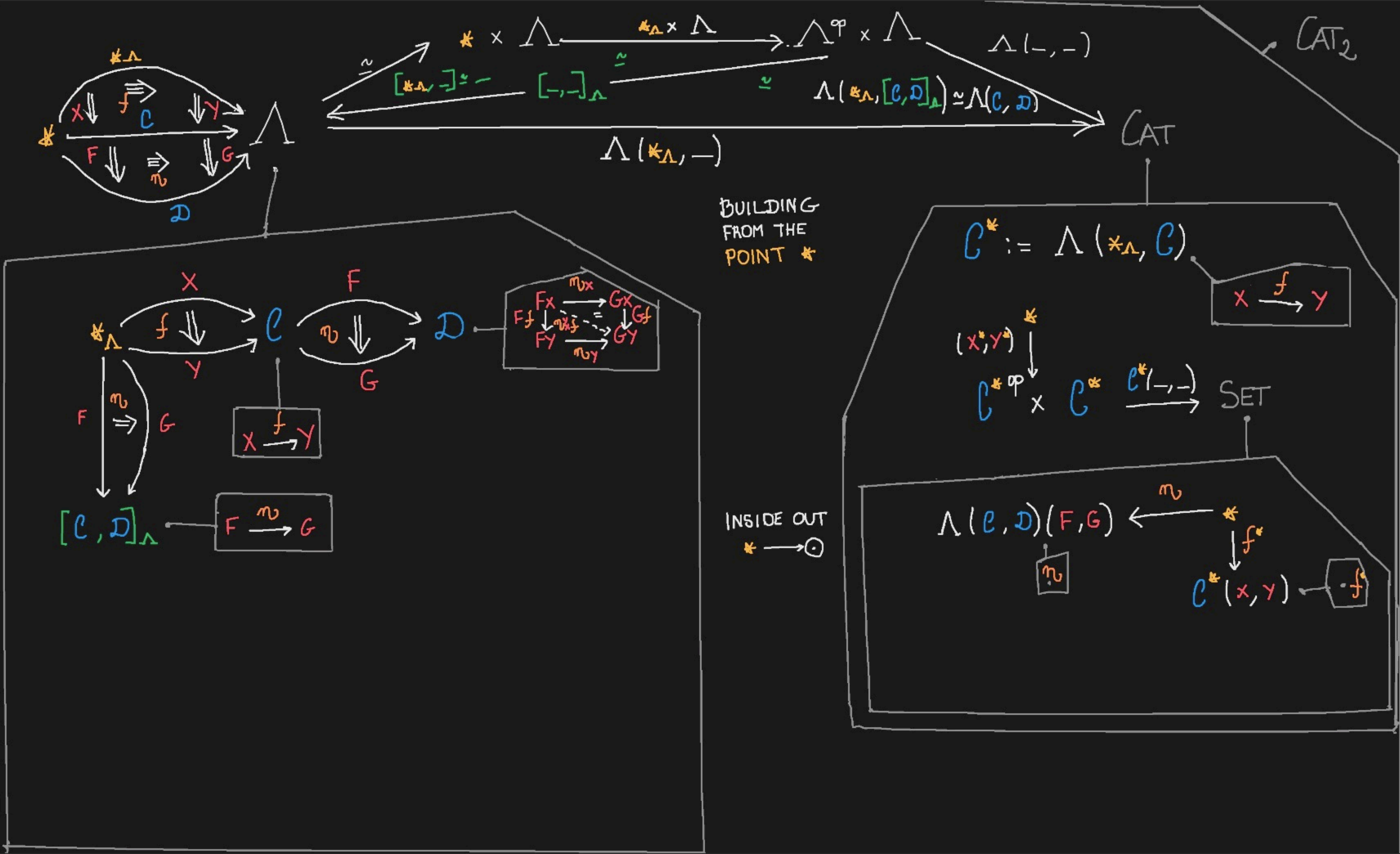


BUILDING FROM THE POINT *
INSIDE OUT
CAT
CAT₂
SET
Λ(*Λ, —)
Λ(—, —)
Λ(*Λ, [C, D]Λ) ≃ Λ(C, D)
C* := Λ(*Λ, C)
C*op × C* → SET
[C, D]Λ

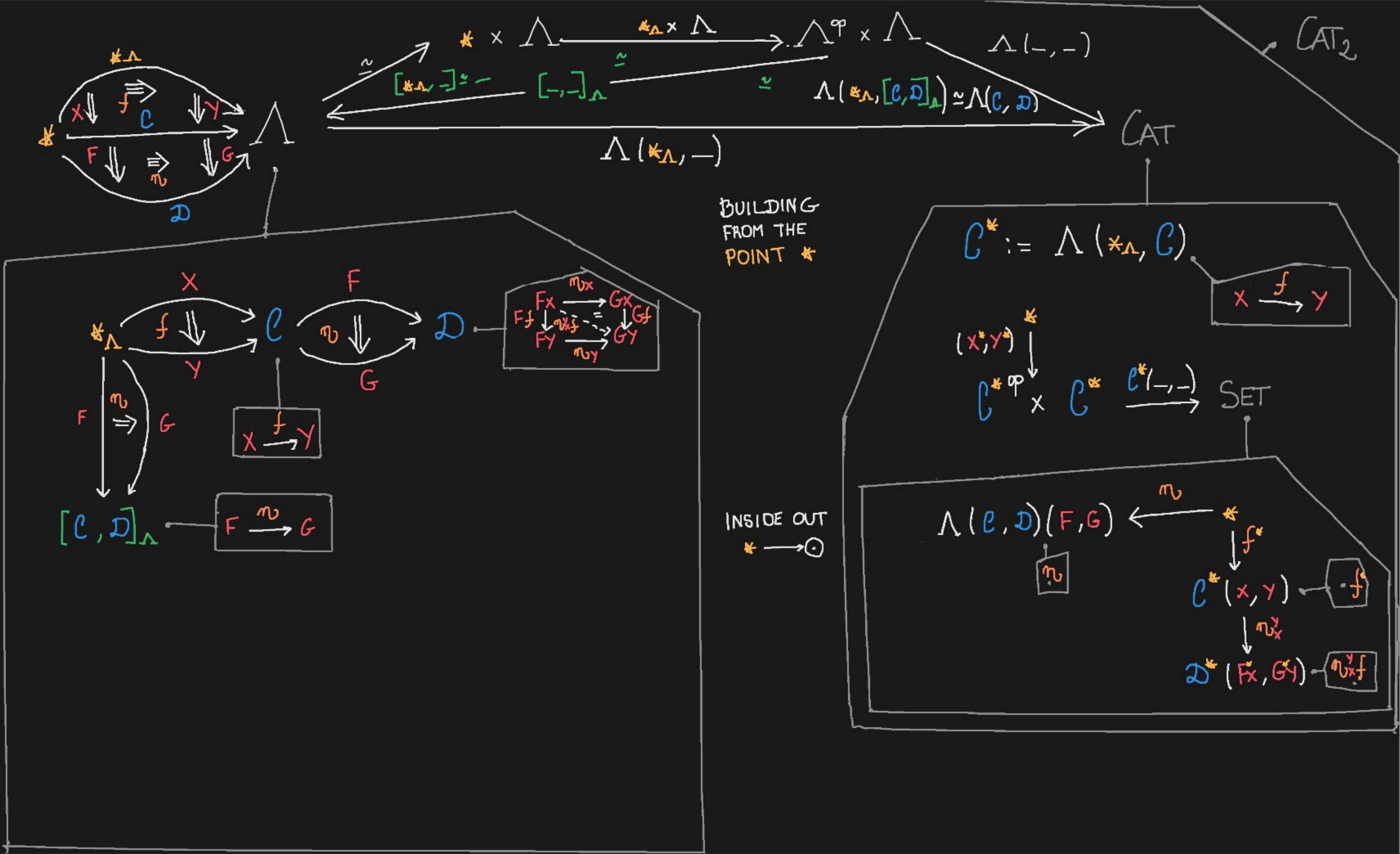


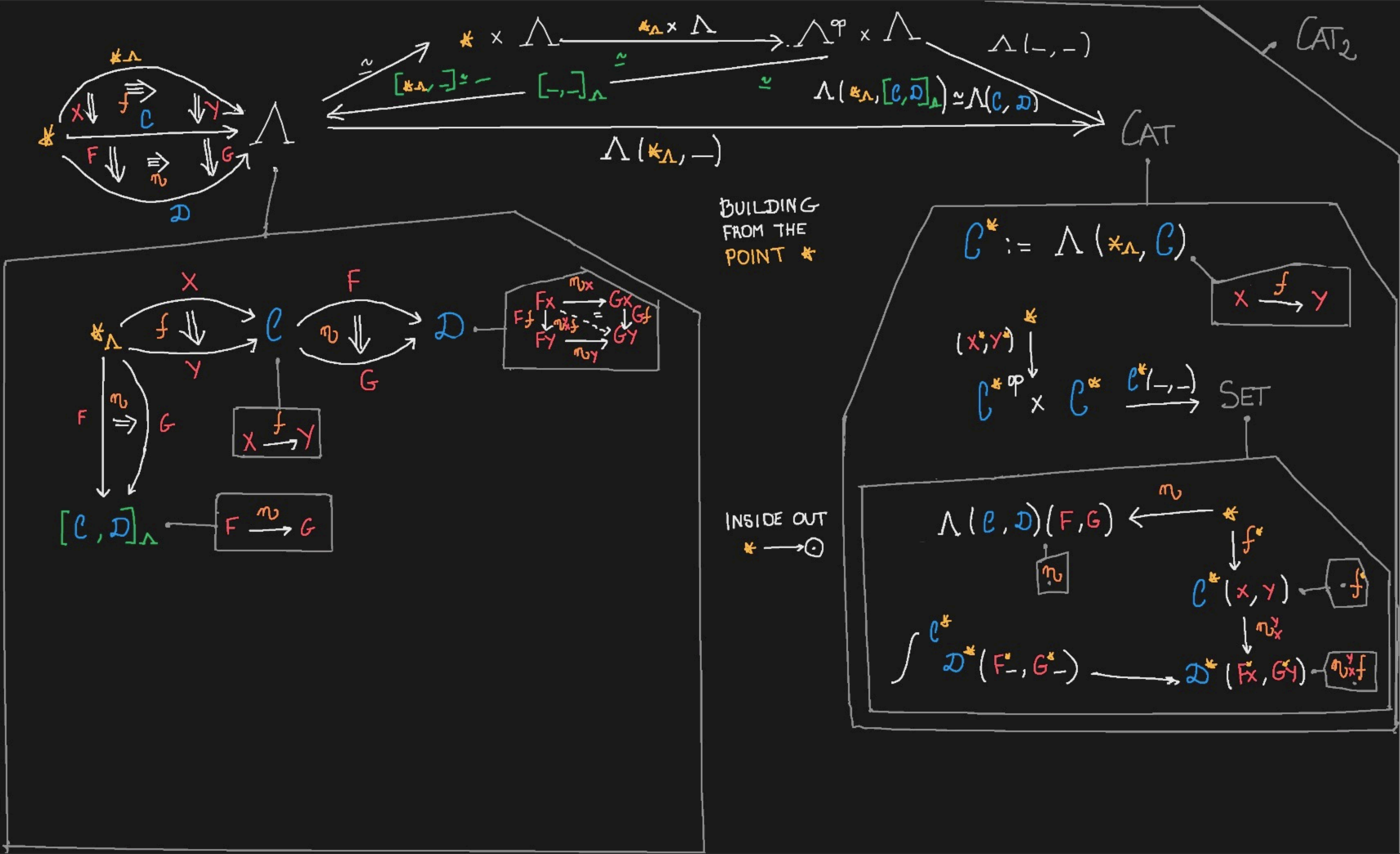
CAT_2
$*_\Lambda \times \Lambda \xrightarrow{*_\Lambda \times \Lambda} \Lambda^{op} \times \Lambda$
$\Lambda(-,-)$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$\Lambda(*_\Lambda, -)$
CAT
BUILDING FROM THE POINT $*$
$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$
$X \xrightarrow{f} Y$
(X^*, Y^*)
$\mathcal{C}^{*op} \times \mathcal{C}^* \xrightarrow{\mathcal{C}^*(-,-)} \mathrm{SET}$
$\mathcal{C}^*(X, Y)$
INSIDE OUT
$* \longrightarrow \odot$
$[\mathcal{C}, \mathcal{D}]_\Lambda$
$F \xrightarrow{\eta} G$
$Fx \xrightarrow{\eta_x} Gx$
Ff, Gf, η_f
$Fy \xrightarrow{\eta_y} Gy$


CAT₂
CAT
Λ
✱ × Λ
✱Λ × Λ
Λ^op × Λ
Λ(–, –)
[✱Λ, –] ≃ –
[–, –]Λ
Λ(✱Λ, [C, D]Λ) ≃ Λ(C, D)
Λ(✱Λ, –)
BUILDING FROM THE POINT ✱
INSIDE OUT
✱ ⟶ ⊙
[C, D]Λ
F ⟶η G
X ⟶f Y
Fx ⟶ηx Gx
Fy ⟶ηy Gy
Ff
Gf
ηf
C✱ := Λ(✱Λ, C)
(x✱, y✱)
C✱op × C✱ ⟶C✱(–, –) SET
C✱(x, y)
f✱

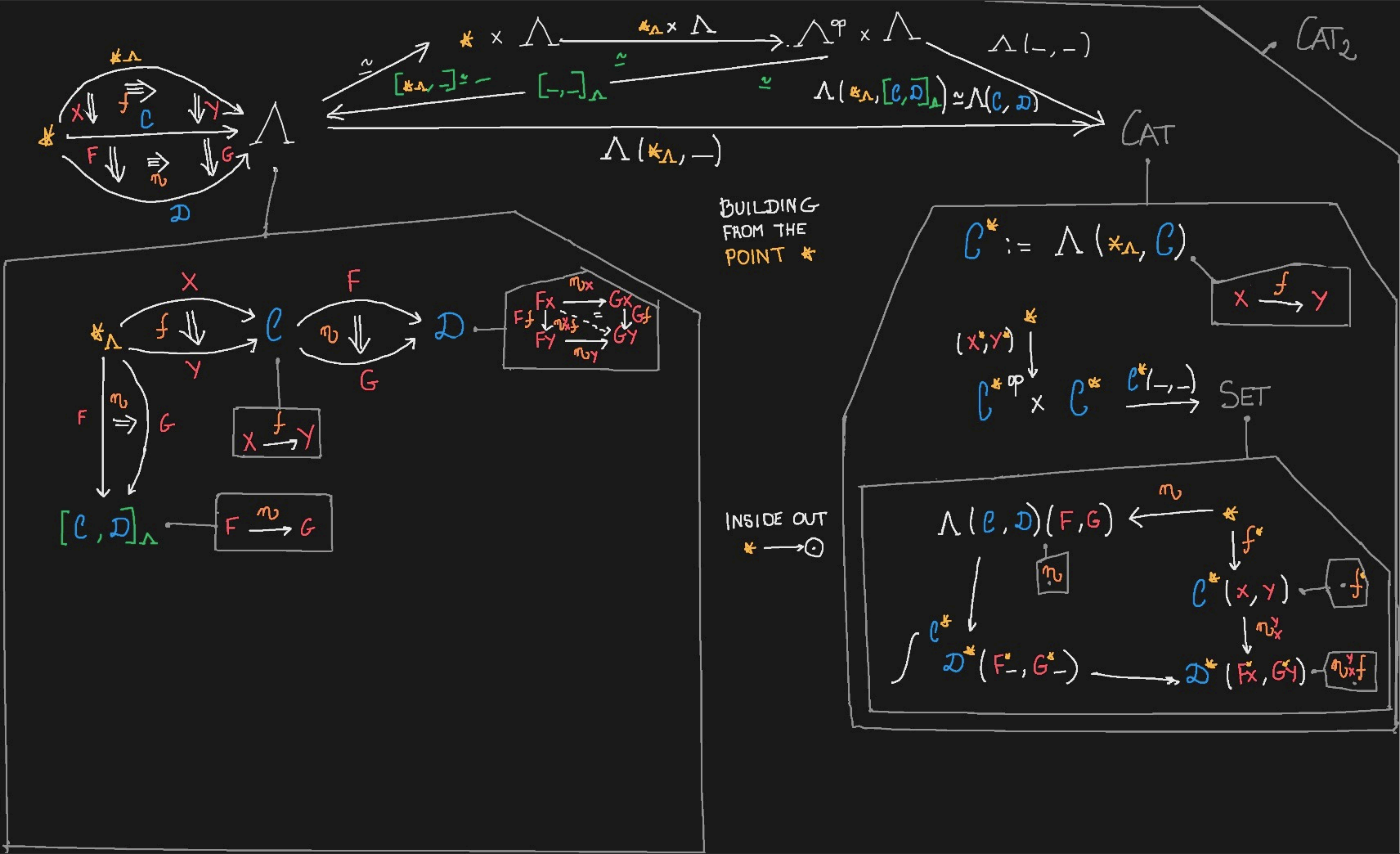


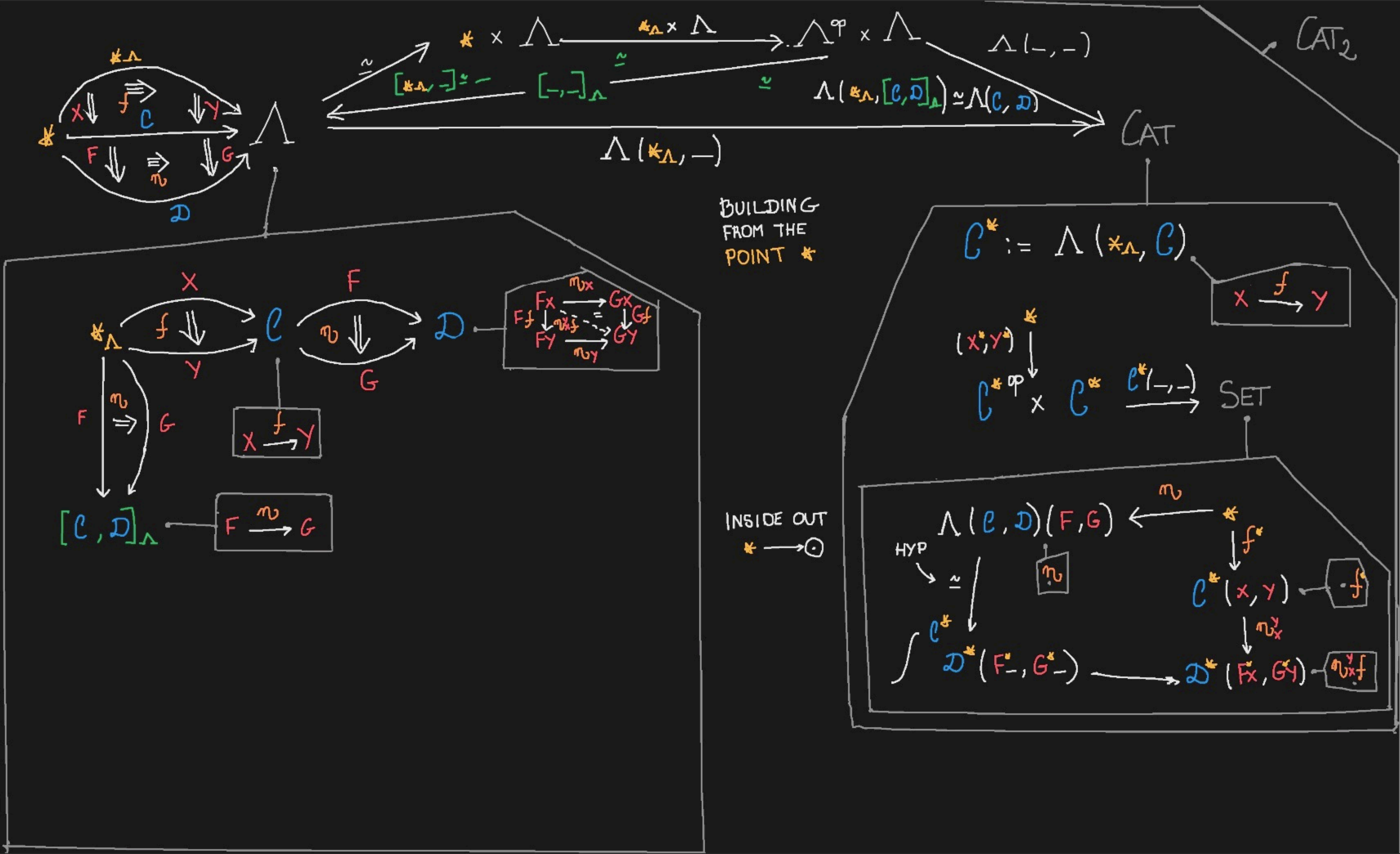
CAT₂
CAT
Λ(*_Λ, −)
Λ(−, −)
*_Λ × Λ
Λ^op × Λ
[*_Λ, −] ≃ −
[−, −]_Λ
Λ(*_Λ, [C, D]_Λ) ≃ Λ(C, D)
BUILDING FROM THE POINT *
INSIDE OUT
* ⟶ ⊙
C* := Λ(*_Λ, C)
X —f→ Y
(X*, Y*)
C*^op × C* —C*(−,−)→ SET
Λ(C, D)(F, G)
C*(X, Y)
[C, D]_Λ
F —η→ G
Fx —ηx→ Gx
Fy —ηy→ Gy
Ff
Gf
ηf


BUILDING FROM THE POINT *
INSIDE OUT
CAT$_2$
CAT
SET
$\Lambda(-,-)$
$\Lambda(*_\Lambda, -)$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$
$\mathcal{C}^{*op} \times \mathcal{C}^* \xrightarrow{\mathcal{C}^*(-,-)} \mathrm{SET}$
$\Lambda(\mathcal{C},\mathcal{D})(F,G)$
$\mathcal{C}^*(x,y)$
$\mathcal{D}^*(Fx, Gy)$
$[\mathcal{C},\mathcal{D}]_\Lambda$

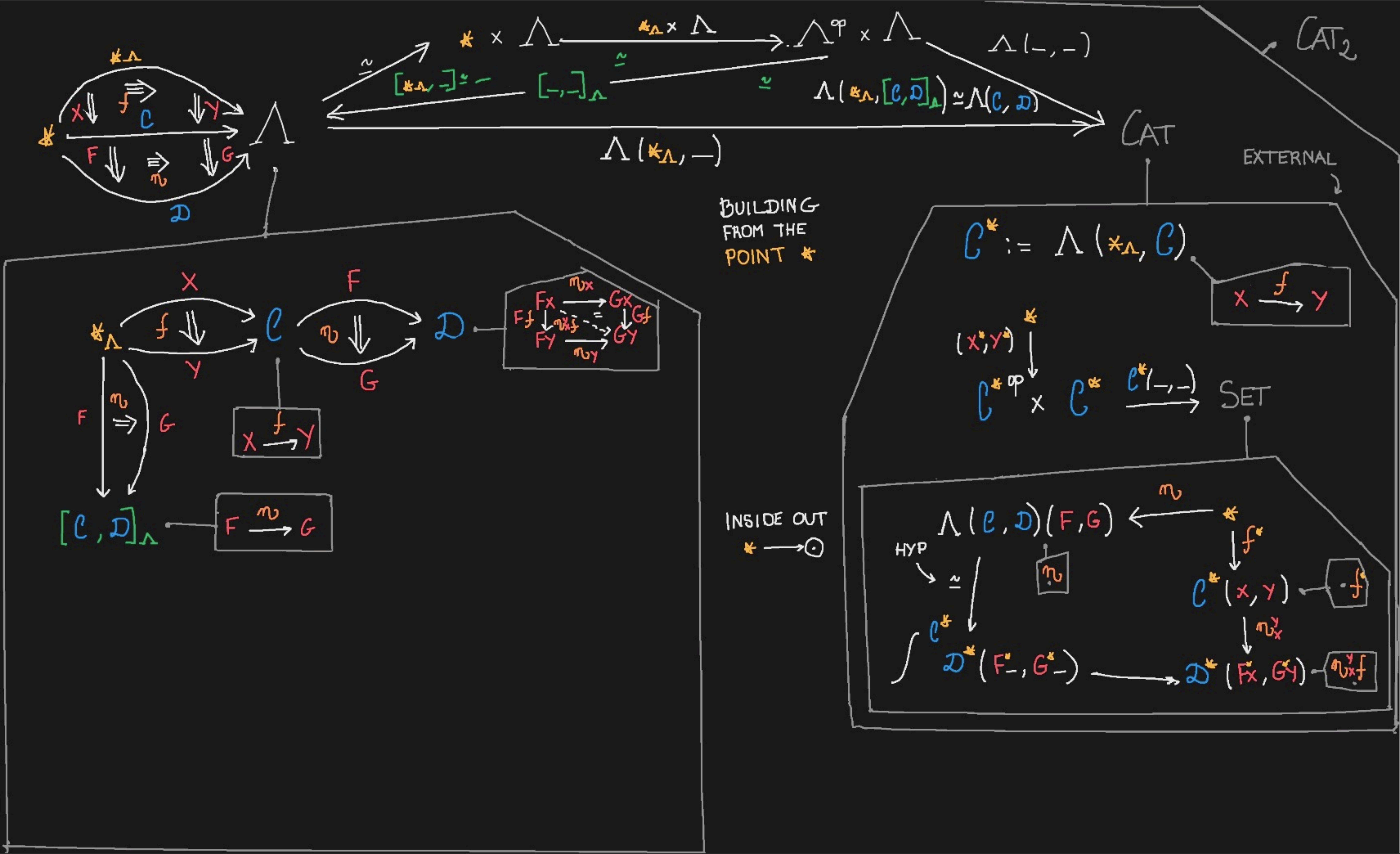


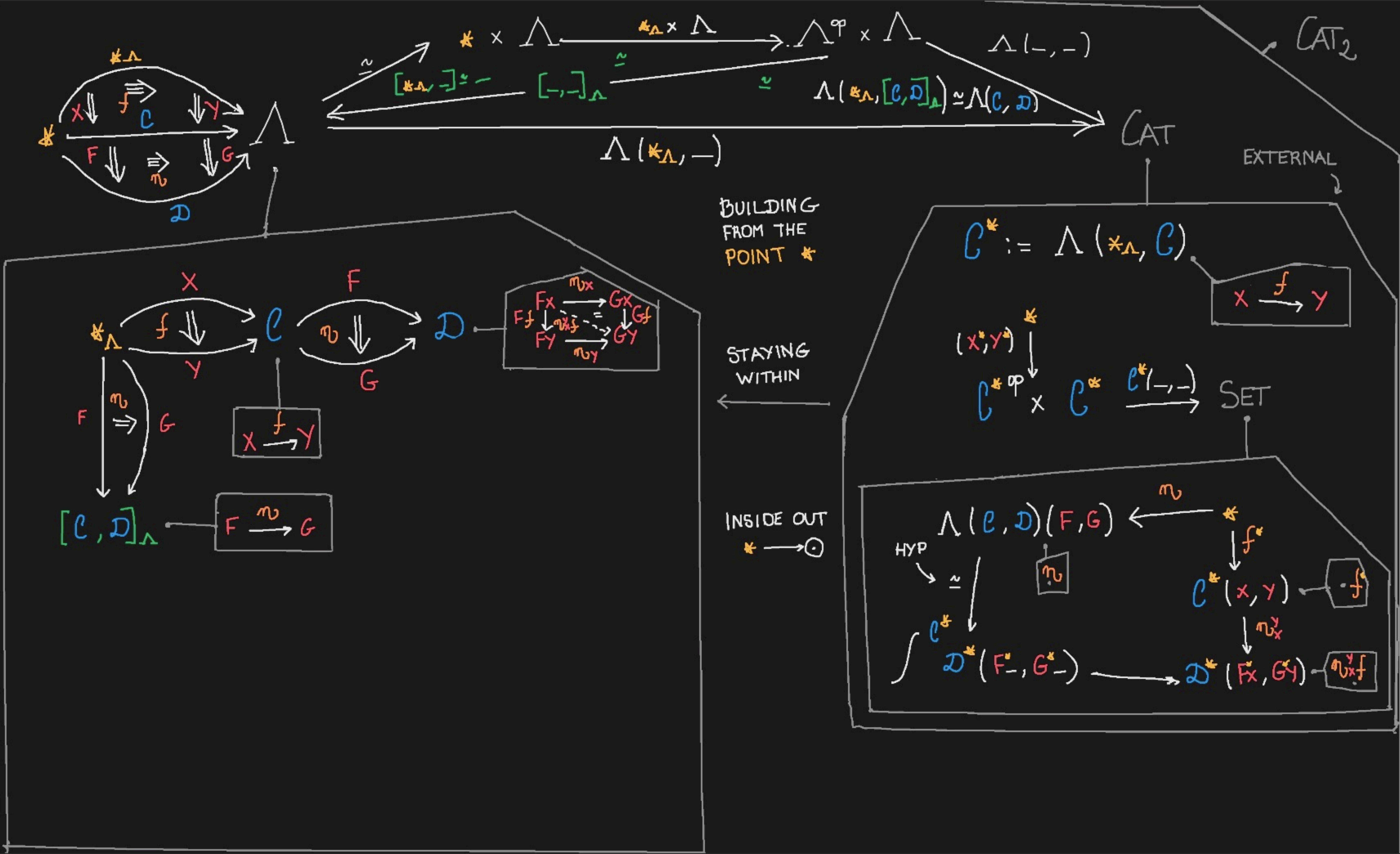
CAT₂
CAT
Λ(−,−)
Λ(*Λ, −)
[*Λ, −] ≃ −
[−,−]Λ
Λ(*Λ, [C,D]Λ) ≃ Λ(C, D)
BUILDING FROM THE POINT *
INSIDE OUT
C* := Λ(*Λ, C)
SET
Λ(C, D)(F, G)
C*(x, y)
D*(Fx, Gy)

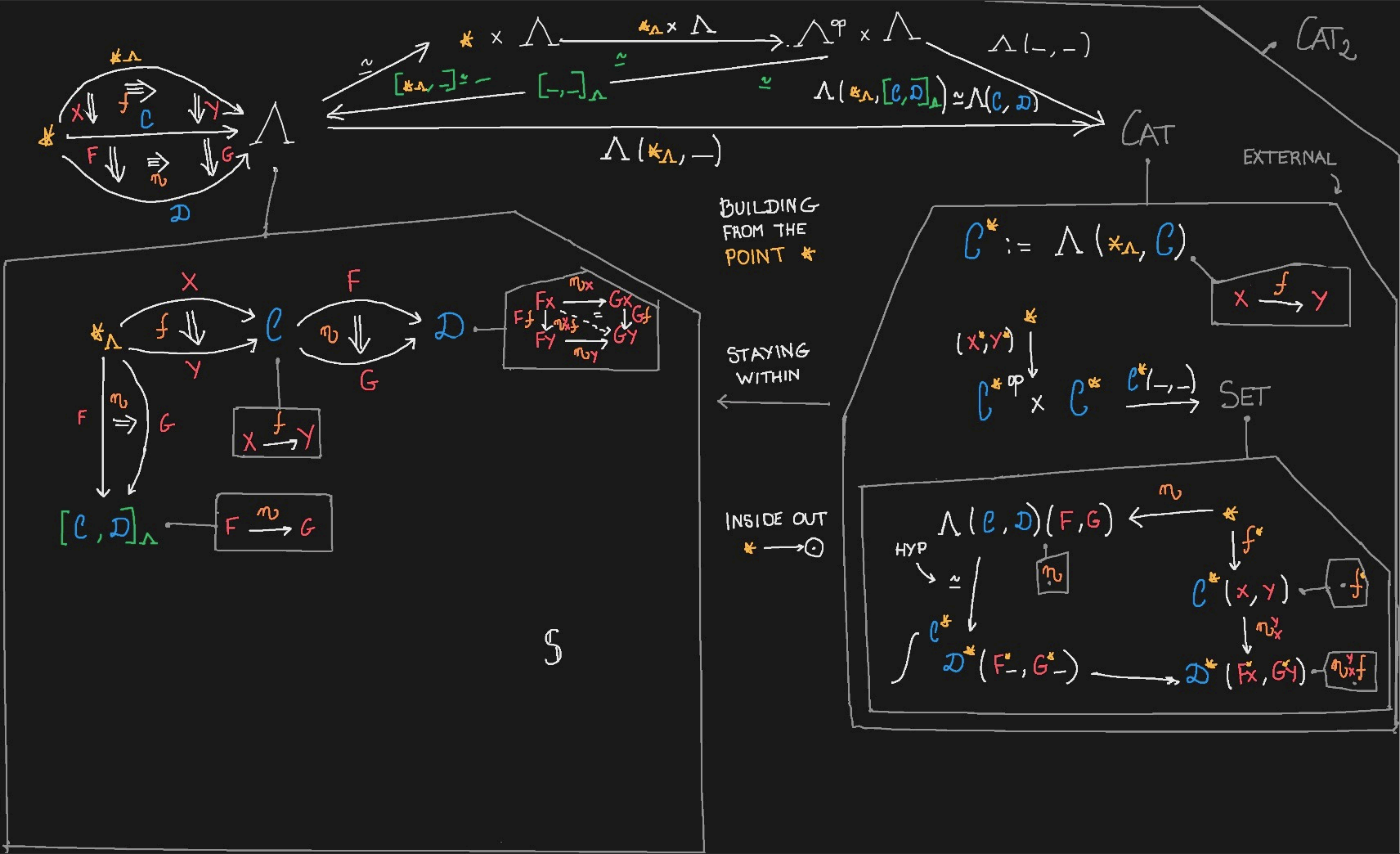

CAT₂
∗_Λ × Λ
∗ × Λ
Λ^op × Λ
Λ(–, –)
[∗_Λ, –] ≃ –
[–, –]_Λ
Λ(∗_Λ, [C, D]_Λ) ≃ Λ(C, D)
CAT
Λ(∗_Λ, –)
BUILDING FROM THE POINT ∗
INSIDE OUT
∗ ⟶ ⊙
C^∗ := Λ(∗_Λ, C)
X →f Y
(x^∗, y^∗)
C^∗op × C^∗ →C^∗(–,–) SET
Λ(C, D)(F, G)
C^∗(x, y)
D^∗(F^∗_, G^∗_)
D^∗(Fx, Gy)
[C, D]_Λ
F →η G
Fx →ηx Gx
Fy →ηy Gy
ηxf
Ff
Gf

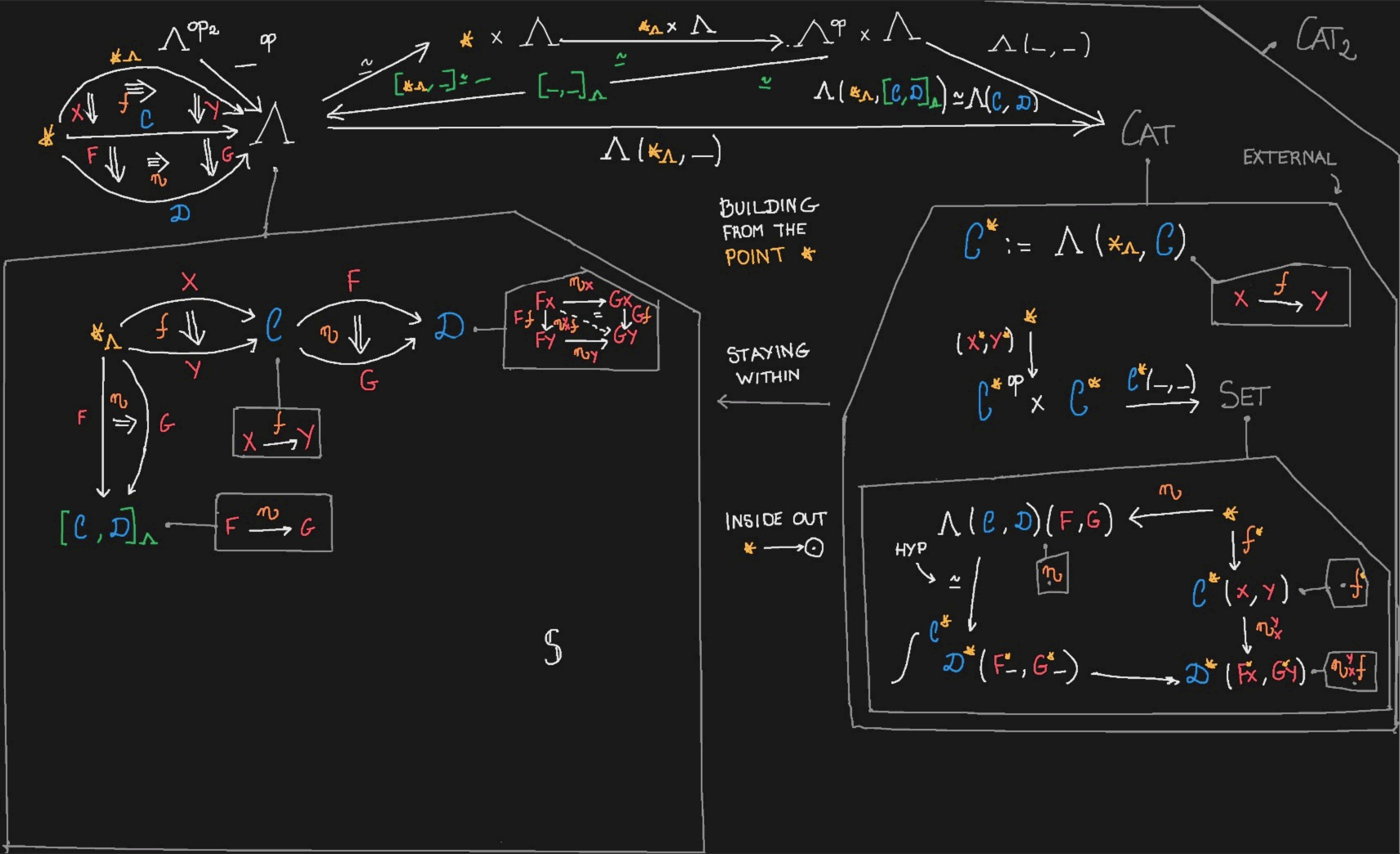


$\mathcal{C}\mathrm{AT}_2$
$* \times \Lambda \xrightarrow{*_\Lambda \times \Lambda} \Lambda^{op} \times \Lambda$
$\Lambda(-,-)$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$\Lambda(*_\Lambda, -)$
$\mathcal{C}\mathrm{AT}$
BUILDING
FROM THE
POINT ✱
INSIDE OUT
$* \longrightarrow \odot$
$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$
$X \xrightarrow{f} Y$
(X^*, Y^*)
$\mathcal{C}^{*op} \times \mathcal{C}^* \xrightarrow{\mathcal{C}^*(-,-)} \mathrm{SET}$
$\Lambda(\mathcal{C},\mathcal{D})(F,G)$
HYP
$\int^{\mathcal{C}^*} \mathcal{D}^*(F^*_-, G^*_-) \longrightarrow \mathcal{D}^*(F^*_X, G^*_Y)$
$\mathcal{C}^*(X,Y)$
η^Y_X
$\eta^Y_X f$
$X \xrightarrow{f} Y$
$[\mathcal{C},\mathcal{D}]_\Lambda$
$F \xrightarrow{\eta} G$
$Fx \xrightarrow{\eta_x} Gx$
$Fy \xrightarrow{\eta_y} Gy$
Ff
Gf
$\eta_x f$

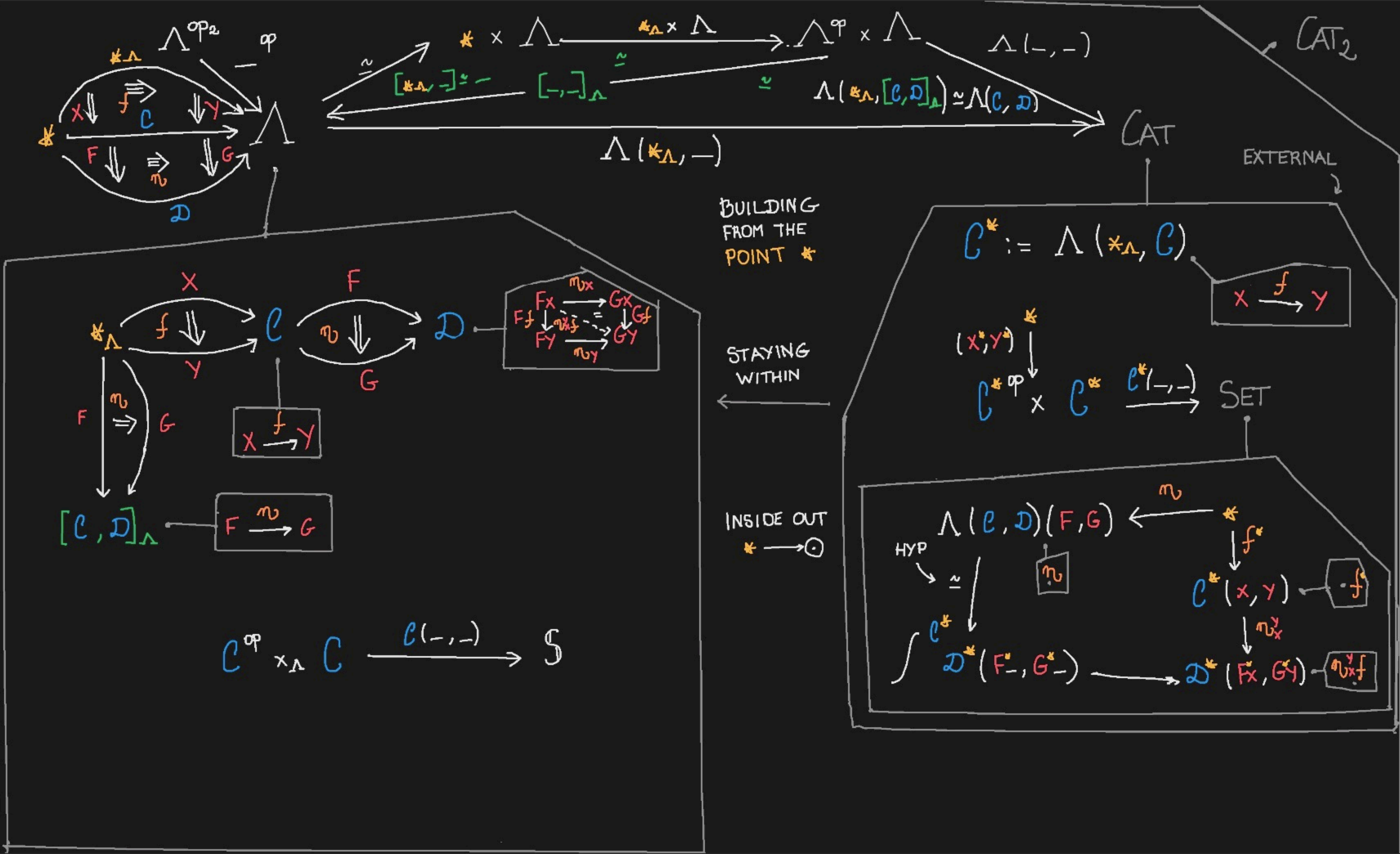

BUILDING FROM THE POINT *
INSIDE OUT
EXTERNAL
CAT₂
CAT
SET
HYP
Λ(*_Λ, —)
Λ(—, —)
Λ(*_Λ, [C, D]_Λ) ≃ Λ(C, D)
C* := Λ(*_Λ, C)
C*^op × C* → SET
Λ(C, D)(F, G)


CAT₂
CAT
EXTERNAL
BUILDING FROM THE POINT
STAYING WITHIN
INSIDE OUT
SET
HYP
Λ(∗_Λ, −)
Λ(−, −)
Λ(∗_Λ, [C, D]_Λ) ≃ Λ(C, D)
C^∗ := Λ(∗_Λ, C)
Λ(C, D)(F, G)

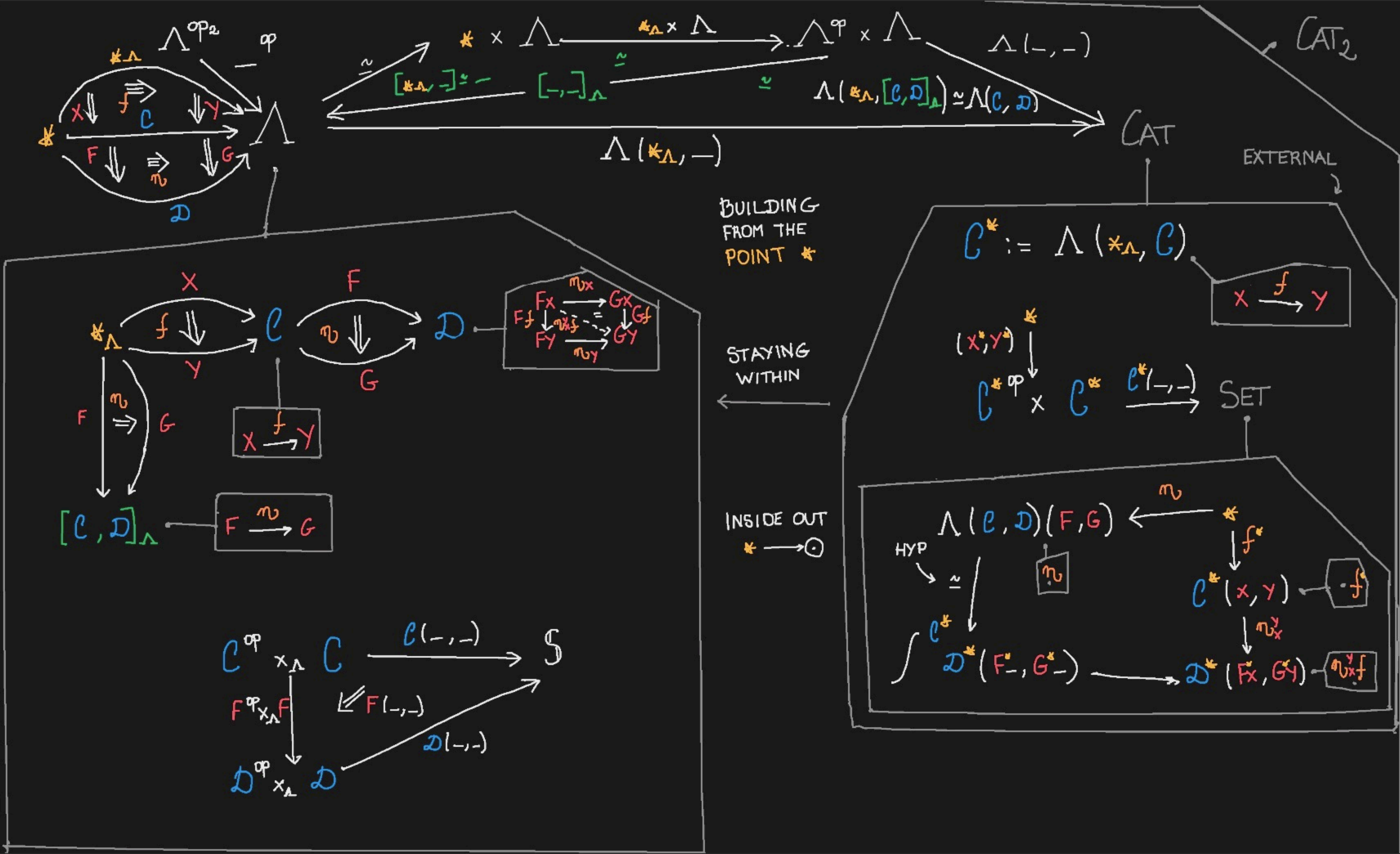

CAT2
CAT
EXTERNAL
BUILDING FROM THE POINT ✱
STAYING WITHIN
INSIDE OUT
✱ ⟶ ⊙
SET
HYP
Λ(✱Λ, —)
Λ(—, —)
✱ × Λ
✱Λ × Λ
Λ^op × Λ
[✱Λ, —] ≃ —
[—, —]Λ
Λ(✱Λ, [C, D]Λ) ≃ Λ(C, D)
C✱ := Λ(✱Λ, C)
X —f→ Y
(X✱, Y✱)
C✱op × C✱ —C✱(—,—)→ SET
F —η→ G
[C, D]Λ
Λ(C, D)(F, G)
C✱(X, Y)
D✱(F✱—, G✱—)
D✱(FX, GY)
∫C✱
ηx
ηy
ηxf
Fx
Gx
Ff
Gf
Fy
Gy
S

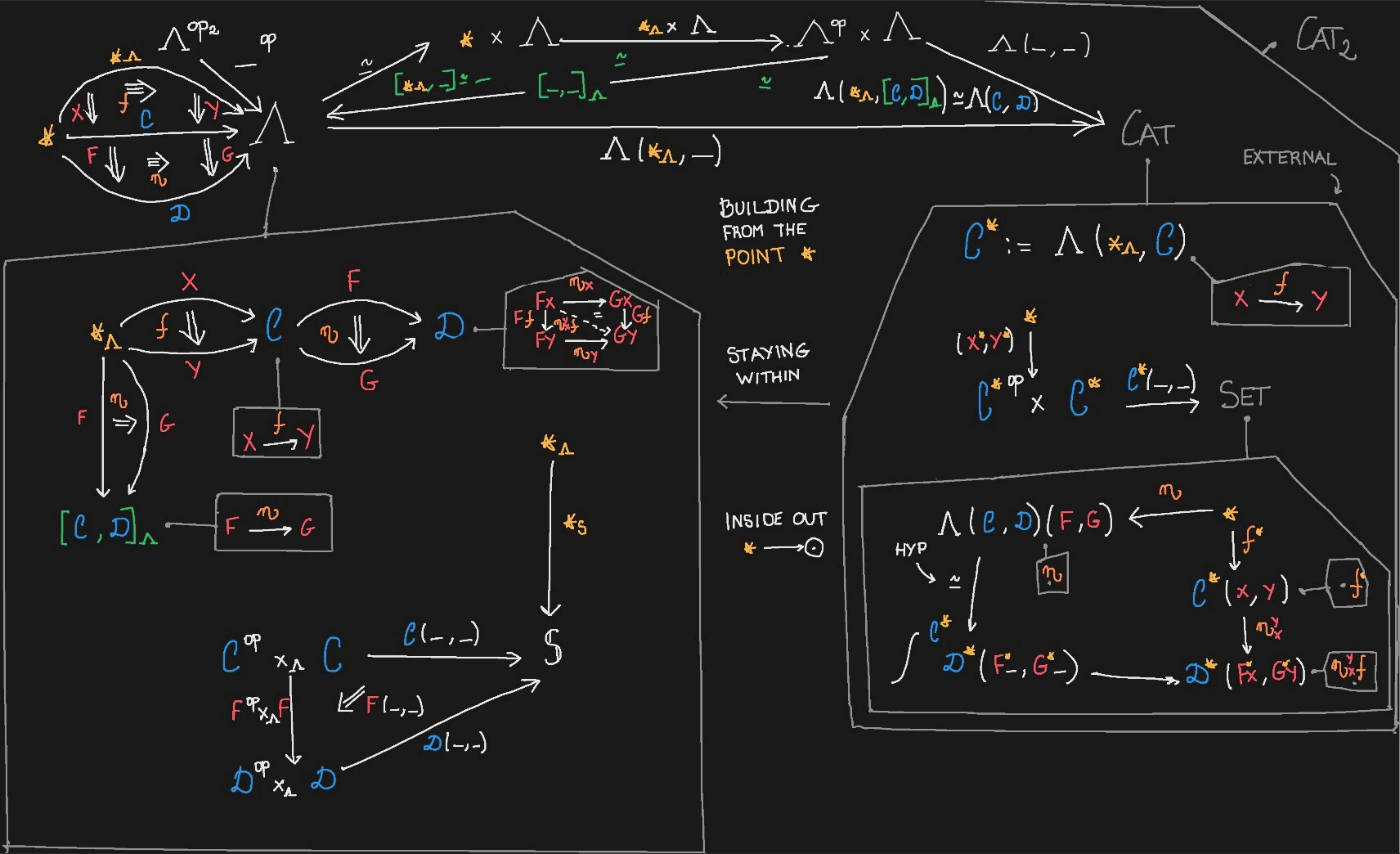


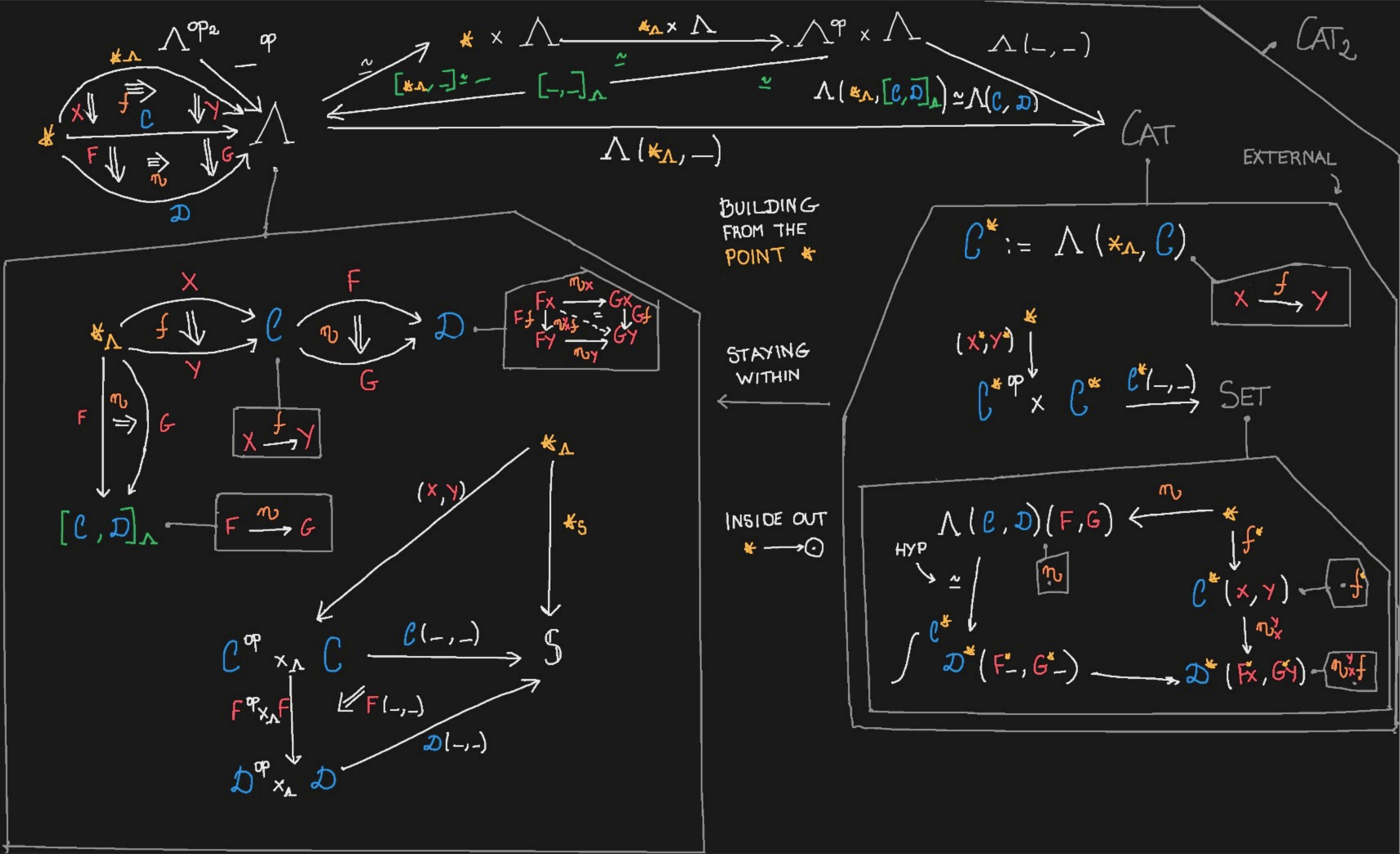
CAT$_2$
CAT
EXTERNAL
Λ^{op_2}
$-^{op}$
$* \times \Lambda \xrightarrow{*_\Lambda \times \Lambda} \Lambda^{op} \times \Lambda$
$\Lambda(-,-)$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$\Lambda(*_\Lambda, -)$
BUILDING FROM THE POINT $*$
STAYING WITHIN
INSIDE OUT
$* \longrightarrow \odot$
$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$
$X \xrightarrow{f} Y$
(X^*, Y^*)
$\mathcal{C}^{*op} \times \mathcal{C}^* \xrightarrow{\mathcal{C}^*(-,-)}$ SET
$\Lambda(\mathcal{C},\mathcal{D})(F,G) \xleftarrow{\eta} *$
HYP
$\simeq$
f^*
$\mathcal{C}^*(X,Y)$
η_X^Y
$\int^{\mathcal{C}^*} \mathcal{D}^*(F^*-, G^*-) \longrightarrow \mathcal{D}^*(F^*X, G^*Y)$
$\eta_X^Y f$
$*_\Lambda \xrightarrow{X, Y} \mathcal{C} \xrightarrow{F, G} \mathcal{D}$
$Fx \xrightarrow{\eta_X} Gx$
Ff
Gf
$\eta_X f$
$Fy \xrightarrow{\eta_Y} Gy$
$[\mathcal{C},\mathcal{D}]_\Lambda$
$F \xrightarrow{\eta} G$
$\mathcal{S}$

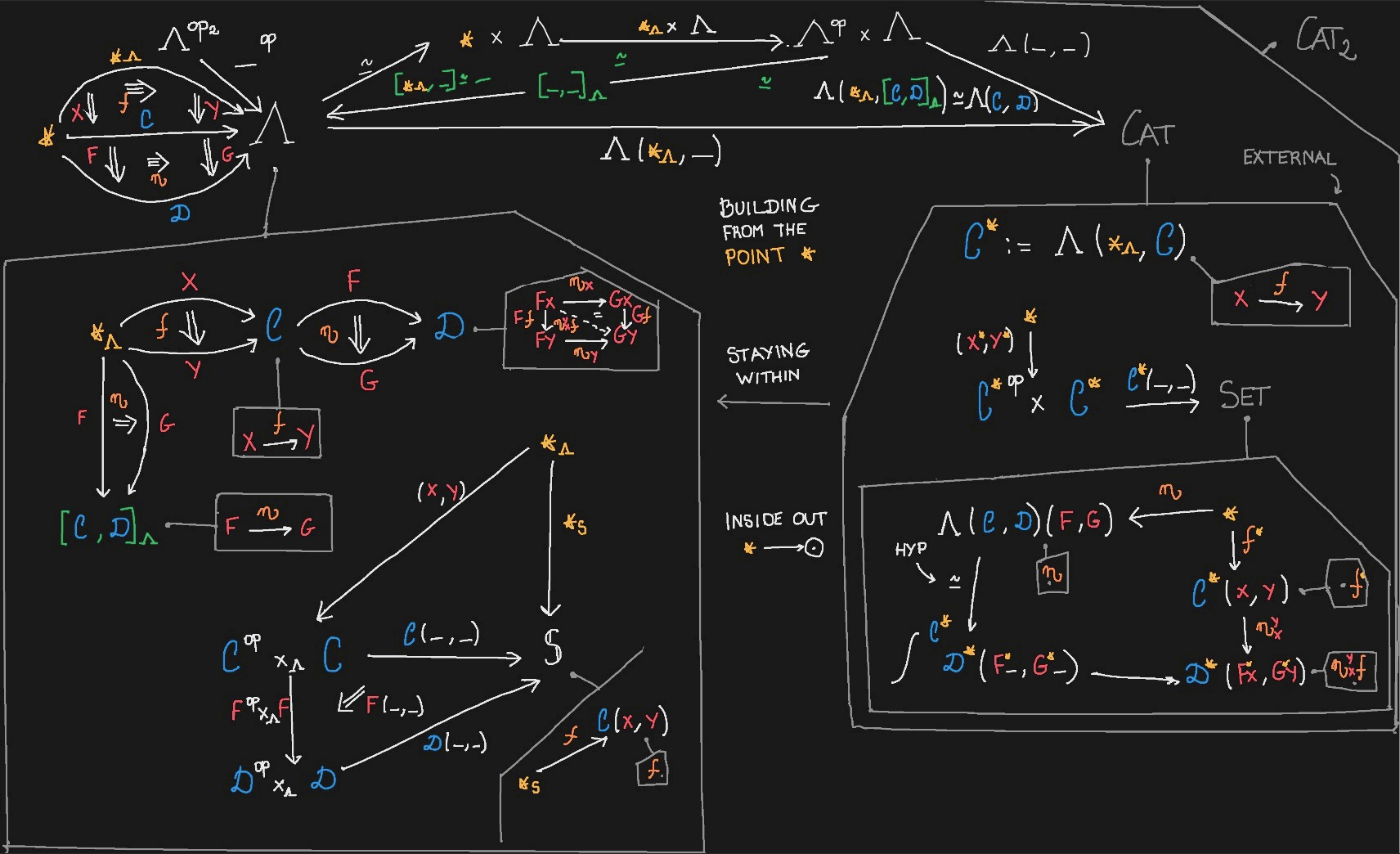


CAT₂
CAT
EXTERNAL
BUILDING FROM THE POINT *
STAYING WITHIN
INSIDE OUT
SET
HYP
Λ(*Λ, —)
Λ(—, —)
Λ(*Λ, [C, D]Λ) ≃ Λ(C, D)
[*Λ, —] ≃ —
[—, —]Λ
C* := Λ(*Λ, C)
C*op × C* → SET
C^op ×Λ C → S
Λ(C, D)(F, G)
D*(F*—, G*—)
D*(Fx, Gy)

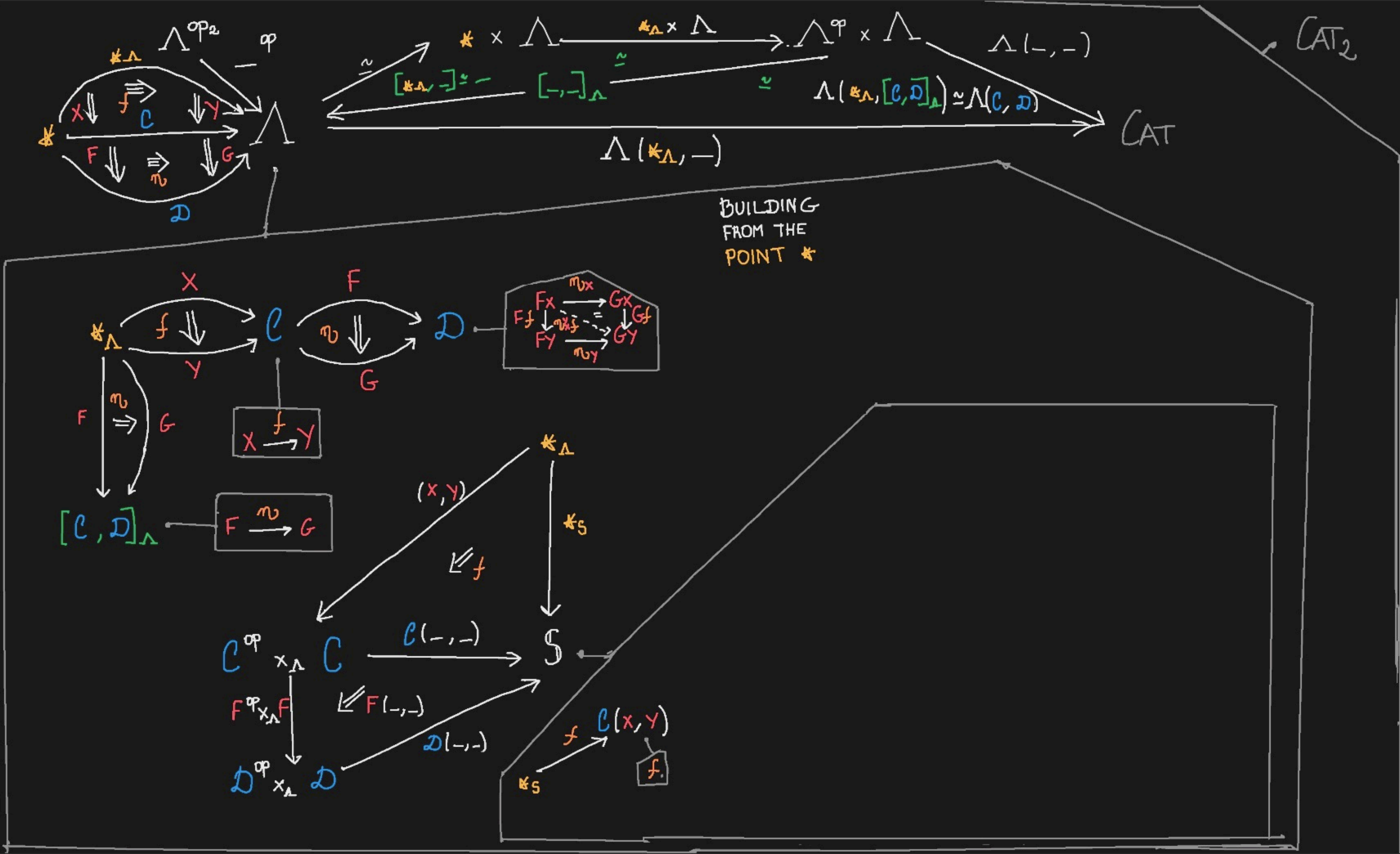

CAT₂
CAT
EXTERNAL
BUILDING FROM THE POINT
STAYING WITHIN
INSIDE OUT
SET
HYP
Λ(–, –)
Λ(*Λ, –)
Λ(*Λ, [C, D]Λ) ≃ Λ(C, D)
C* := Λ(*Λ, C)
[C, D]Λ
Λ(C, D)(F, G)
C*(x, y)
D*(Fx, Gy)

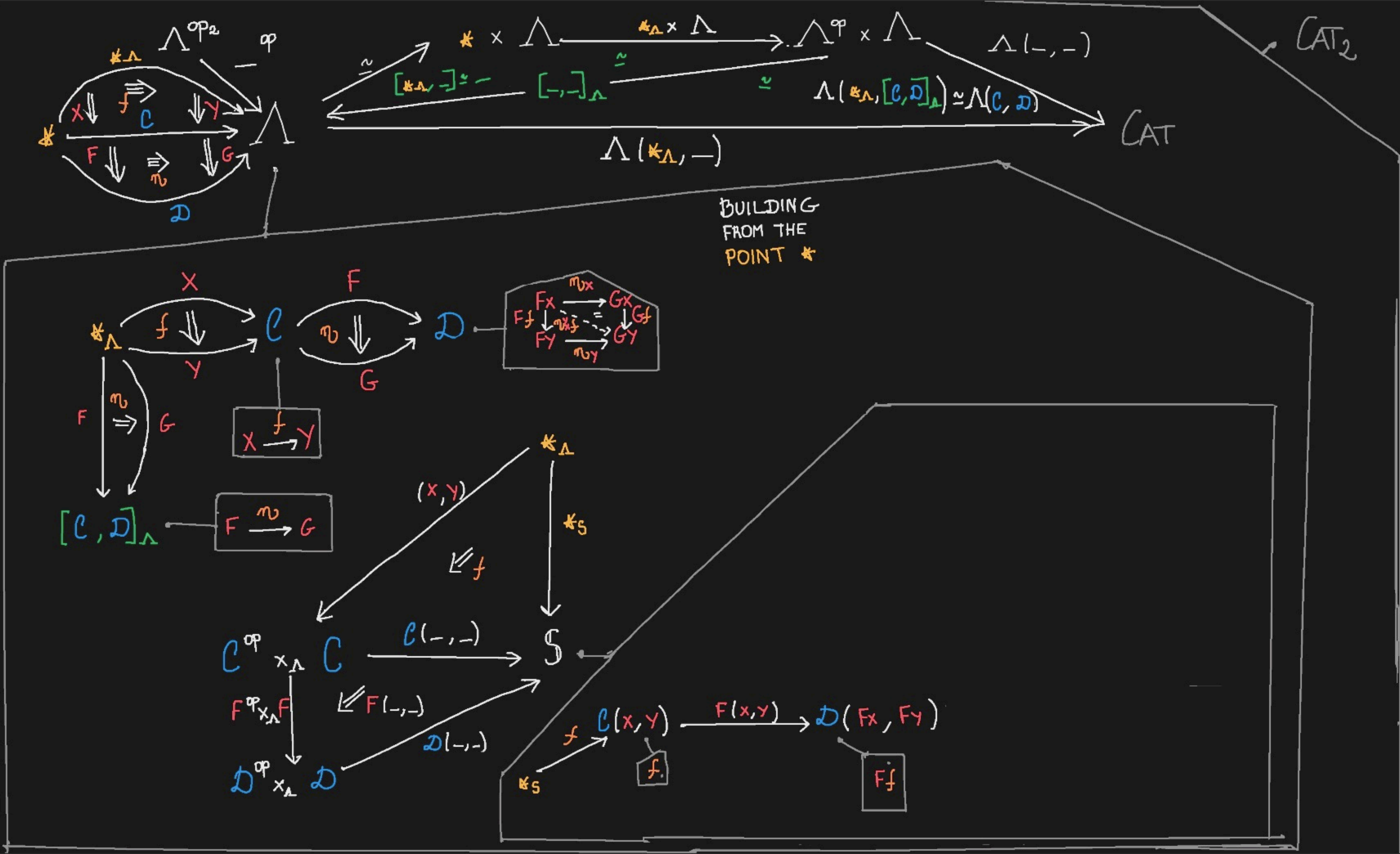

CAT2
CAT
EXTERNAL
BUILDING FROM THE POINT
STAYING WITHIN
INSIDE OUT
SET
HYP


CAT2
CAT
EXTERNAL
BUILDING
FROM THE
POINT
STAYING
WITHIN
INSIDE OUT
SET
HYP

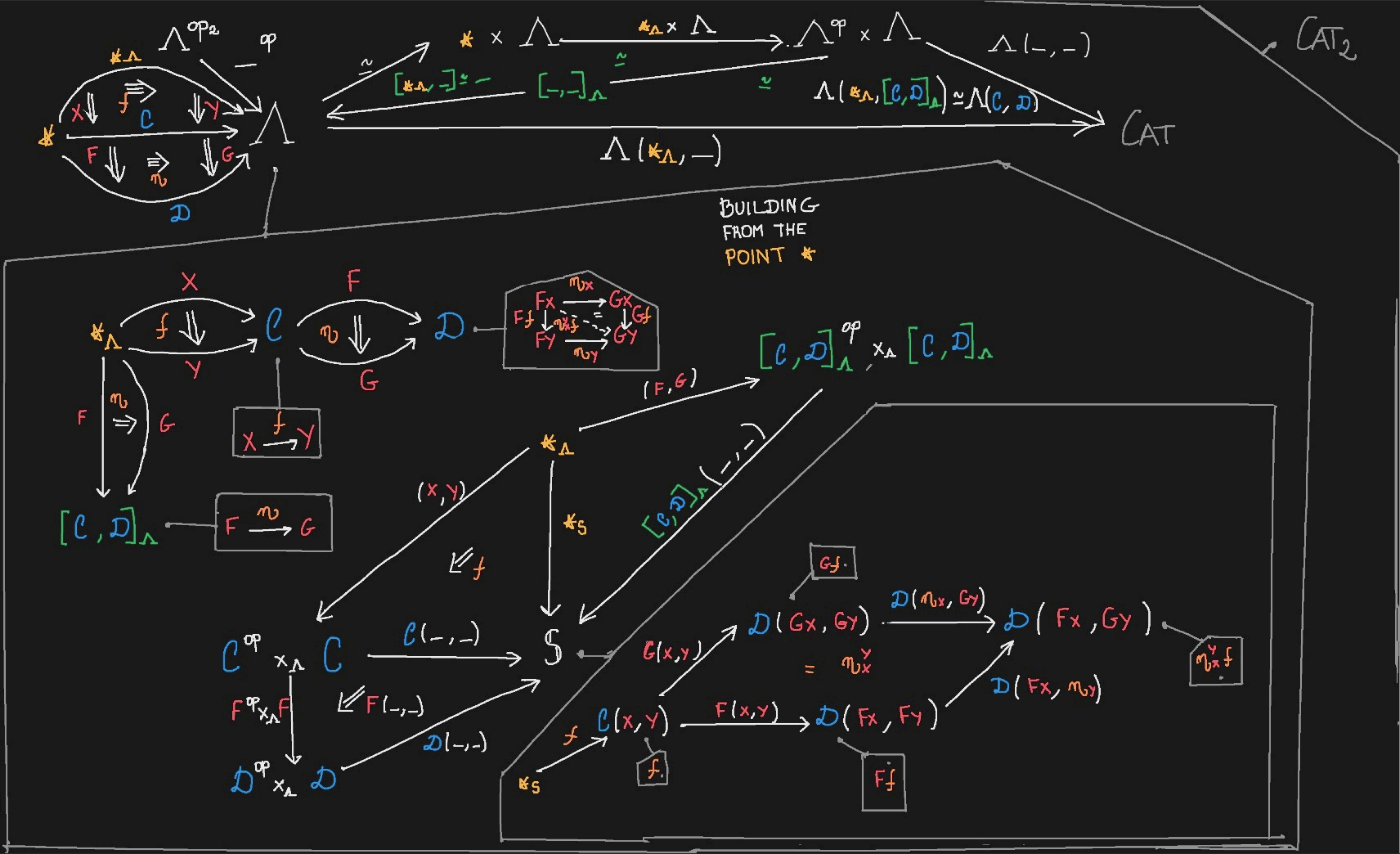


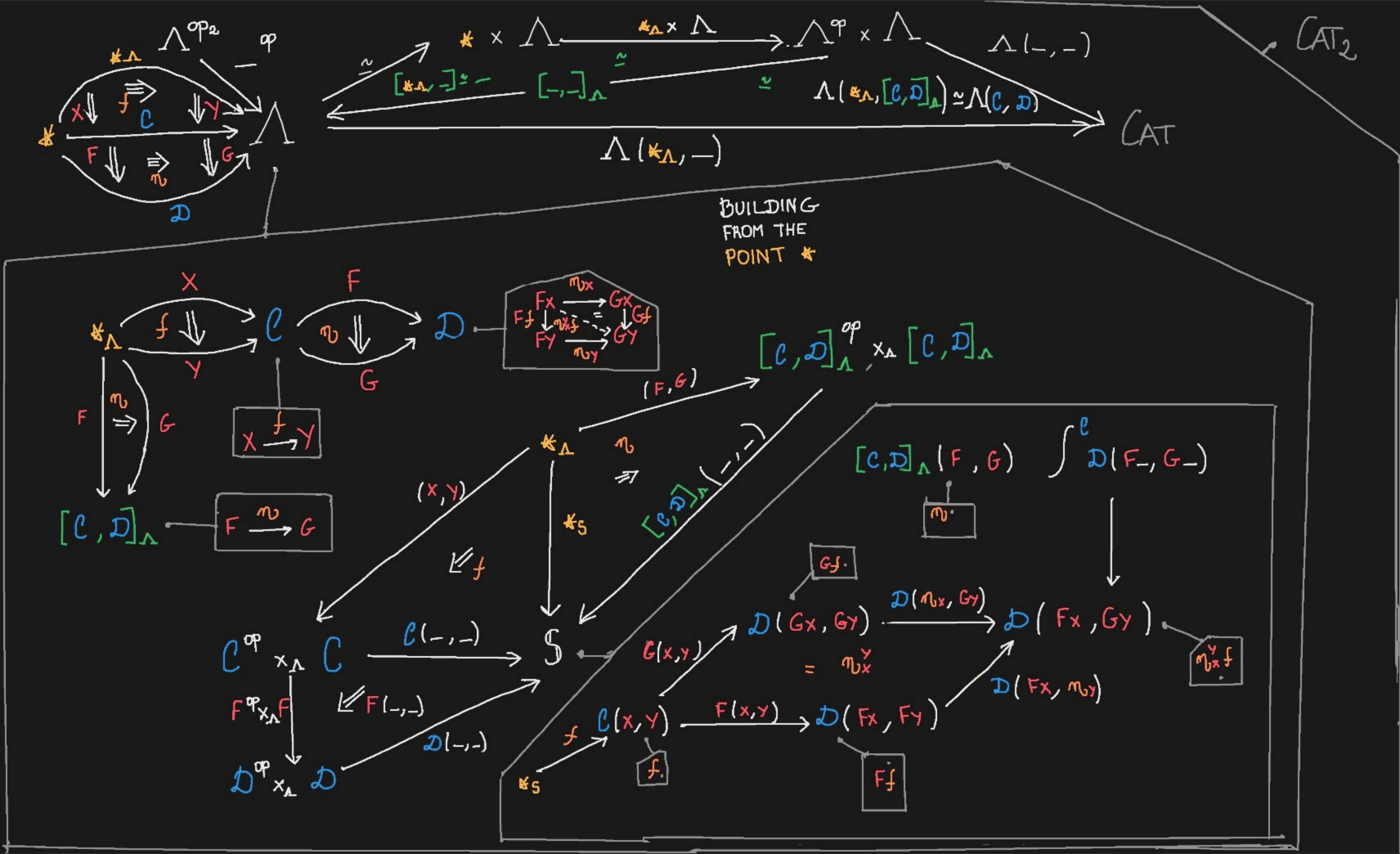
CAT₂
CAT
EXTERNAL
BUILDING FROM THE POINT *
STAYING WITHIN
INSIDE OUT
* ⟶ ⊙
Λ^{op_2}
$\Lambda(*_\Lambda, -)$
$\Lambda(-,-)$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$[\mathcal{C},\mathcal{D}]_\Lambda$
$\mathcal{C}^* := \Lambda(*_\Lambda, \mathcal{C})$
$\mathcal{C}^{*op} \times \mathcal{C}^* \xrightarrow{\mathcal{C}^*(-,-)}$ SET
$\Lambda(\mathcal{C},\mathcal{D})(F,G)$
HYP
$\mathcal{C}^*(x,y)$
$\mathcal{D}^*(F^*_-, G^*_-)$
$\mathcal{D}^*(Fx, Gy)$
$\mathcal{C}^{op} \times_\Lambda \mathcal{C} \xrightarrow{\mathcal{C}(-,-)} \mathcal{S}$
$F(-,-)$
$\mathcal{D}(-,-)$
$\mathcal{D}^{op} \times_\Lambda \mathcal{D}$
$\mathcal{C}(x,y)$


CAT₂
CAT
BUILDING
FROM THE
POINT *
Λ(–, –)
Λ(*_Λ, –)
Λ(*_Λ, [C, D]_Λ) ≃ Λ(C, D)
[–, –]_Λ
[*_Λ, –] ≃ –
*_Λ × Λ
Λ^op × Λ
Λ^op₂
–^op
X → Y
F → G
C(–, –)
D(–, –)
F(–, –)
F^op ×_Λ F
C^op ×_Λ C
D^op ×_Λ D
(X, Y)
*_S
C(X, Y)
f.

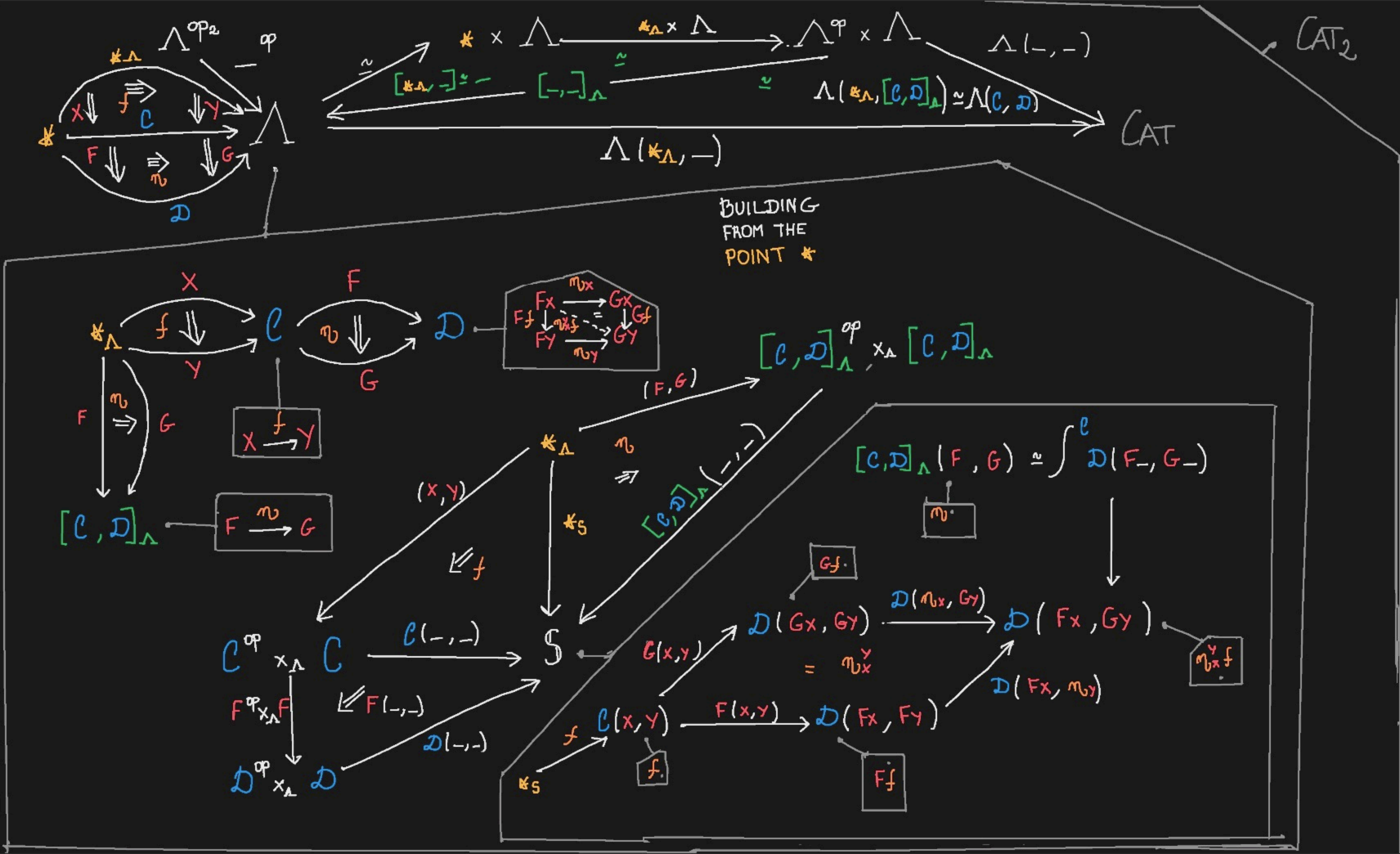


Λ^{op_2}
$-^{op}$
$*_\Lambda$
$\mathcal{C}$
$\mathcal{D}$
Λ
$* \times \Lambda \xrightarrow{*_\Lambda \times \Lambda} \Lambda^{op} \times \Lambda$
$[*_\Lambda, -] \simeq -$
$[-,-]_\Lambda$
$\Lambda(-,-)$
$\Lambda(*_\Lambda, [\mathcal{C},\mathcal{D}]_\Lambda) \simeq \Lambda(\mathcal{C},\mathcal{D})$
$\Lambda(*_\Lambda, -)$
CAT
CAT_2
BUILDING
FROM THE
POINT *
$Fx \xrightarrow{\eta_x} Gx$
$Fy \xrightarrow{\eta_y} Gy$
Ff
Gf
η_f
$x \xrightarrow{f} y$
$[\mathcal{C},\mathcal{D}]_\Lambda$
$F \xrightarrow{\eta} G$
(x,y)
$*_S$
$\mathcal{C}^{op} \times_\Lambda \mathcal{C} \xrightarrow{\mathcal{C}(-,-)} S$
$F^{op} \times_\Lambda F$
$F(-,-)$
$\mathcal{D}^{op} \times_\Lambda \mathcal{D}$
$\mathcal{D}(-,-)$
$\mathcal{C}(x,y) \xrightarrow{F(x,y)} \mathcal{D}(Fx,Fy)$
f
Ff


CAT₂
CAT
BUILDING FROM THE POINT *
Λ(*Λ, [C, D]Λ) ≃ Λ(C, D)
Λ(*Λ, —)
Λ(—, —)
[*Λ, —] ≃ —
[—, —]Λ
[C, D]Λ
(F, G)
(X, Y)
C(—, —)
D(—, —)
F(—, —)
D(Gx, Gy)
D(Fx, Gy)
D(Fx, Fy)
C(x, y)
D(ηx, Gy)
D(Fx, ηy)
G(x, y)
F(x, y)



CAT₂
CAT
Λ(−,−)
Λ(∗Λ,−)
Λ(∗Λ,[C,D]Λ) ≃ Λ(C,D)
[∗Λ,−] ≃ −
[−,−]Λ
BUILDING
FROM THE
POINT ∗
[C,D]Λ
[C,D]Λ^op ×Λ [C,D]Λ
(F,G)
(X,Y)
C(−,−)
F(−,−)
D(−,−)
C^op ×Λ C
D^op ×Λ D
F^op ×Λ F
[C,D]Λ(F,G)
∫^C D(F−,G−)
D(Gx,Gy)
D(Fx,Gy)
D(Fx,Fy)
C(x,y)
D(ηx,Gy)
D(Fx,ηy)
F(x,y)
G(x,y)
= η_x^y



CAT₂
CAT
Λ(–, –)
Λ(*_Λ, [C, D]_Λ) ≃ Λ(C, D)
Λ(*_Λ, –)
[*_Λ, –] ≃ –
[–, –]_Λ
BUILDING
FROM THE
POINT *
[C, D]_Λ
[C, D]_Λ^op ×_Λ [C, D]_Λ
C^op ×_Λ C
D^op ×_Λ D
C(–, –)
D(–, –)
F(–, –)
[C, D]_Λ(F, G) ≃ ∫^C D(F–, G–)
D(Gx, Gy)
D(ηx, Gy)
D(Fx, Gy)
D(Fx, ηy)
C(x, y)
F(x, y)
D(Fx, Fy)
G(x, y)

$$\begin{tikzcd}
\Lambda^{op_2} \arrow[dr, "(-)^{op}"] & \\
* \arrow[r, bend left, "*_\Lambda"] \arrow[r, bend right, "\mathbb{S}"'] & \Lambda
\end{tikzcd}$$

$$\begin{tikzcd}
 & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] \arrow[dll, "{[-,-]_\Lambda}"'] & \\
\Lambda \arrow[ur, "\cong"] \arrow[rrr, "{\Lambda(*_\Lambda, -)}"'] & & & \mathcal{CAT}
\end{tikzcd}$$

$[*_\Lambda, -] \cong -$ $\qquad \cong \qquad \cong$

$\Lambda(*_\Lambda, [-,-]_\Lambda) \cong \Lambda(-,-)$

$\mathcal{CAT}_2$

BUILDING
FROM THE
POINT $*$

$$\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda \arrow[rr, "{\mathcal{C}(-,-)_* \times Id}"] \arrow[d, "Id_*"'] \arrow[dr, phantom, "\text{YONEDA} \cong"] & & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[dll, "{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda(-,-)}"] \\
\mathbb{S} & &
\end{tikzcd}$$

$$\begin{tikzcd}
* \arrow[r, bend left, "*_\Lambda"] \arrow[r, bend right, "S"'] & \Lambda & \Lambda^{op_2} \arrow[l, "-^{op}"']
\end{tikzcd}$$

$$\begin{tikzcd}
\Lambda \arrow[r, "\simeq"] \arrow[rrr, bend right, "\Lambda(*_\Lambda, -)"'] & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] \arrow[ll, "{[-,-]_\Lambda}"'] & CAT
\end{tikzcd}$$

$[*_\Lambda, -] \simeq -$

$\simeq$

$\Lambda(*_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

CAT_2

BUILDING FROM THE POINT $*$

$$\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, S]_\Lambda \arrow[r, "\mathcal{C}(-,-)_* \times Id"] \arrow[d, "Id_*"'] \arrow[dr, phantom, "\text{YONEDA} \simeq"] & {[\mathcal{C}^{op}, S]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, S]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, S]_\Lambda(-,-)}"] \\
S &
\end{tikzcd}$$

$$*_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, S]_\Lambda$$

F

$$
\begin{tikzcd}
\ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "\mathbb{S}"'] & \Lambda & \Lambda^{op_2} \arrow[l, "(-)^{op}"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] \arrow[dll, "{[-,-]_\Lambda}"'] & \\
\Lambda \arrow[ur, "\simeq"] \arrow[rrr, "{\Lambda(\ast_\Lambda, -)}"'] & & & CAT
\end{tikzcd}
$$

$[\ast_\Lambda, -] \simeq -$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

CAT_2

BUILDING FROM THE POINT $\ast$

$$
\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda \arrow[r, "{\mathcal{C}(-,-)_\ast \times Id}"] \arrow[d, "Id_\ast"'] \arrow[dr, phantom, "\text{YONEDA} \simeq"] & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda(-,-)}"] \\
\mathbb{S} &
\end{tikzcd}
$$

$$
\ast_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda
$$

$$
\boxed{\begin{aligned} F_\bullet &\simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,\bullet), F_-) \\ &\simeq [\mathcal{C}^{op}, \mathbb{S}]_\Lambda(\mathcal{C}(-,\bullet), F_-) \end{aligned}}
$$

$$
\begin{tikzcd}
\ast \arrow[rr, bend left, "\ast_\Lambda"] \arrow[rr, bend right, "S"'] & & \Lambda & \Lambda^{op_2} \arrow[l, "-^{op}"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
\Lambda \arrow[r, "\simeq"] \arrow[rrr, bend right, "\Lambda(\ast_\Lambda, -)"'] & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] \arrow[ll, bend right, "{[-,-]_\Lambda}"'] & \mathrm{CAT}
\end{tikzcd}
$$

$[\ast_\Lambda, -] \simeq -$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

CAT_2

BUILDING
FROM THE
POINT $\ast$

$$
\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, S]_\Lambda \arrow[r, "\mathcal{C}(-,-)_\ast \times Id"] \arrow[d, "Id_\ast"'] \arrow[dr, phantom, "\overset{\text{YONEDA}}{\simeq}"] & {[\mathcal{C}^{op}, S]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, S]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, S]_\Lambda(-,-)}"] \\
S &
\end{tikzcd}
$$

$$Fx \simeq \int^{\mathcal{C}^{op}} S(\mathcal{C}(-,x), F)$$

$$\ast_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, S]_\Lambda$$

$$F_\bullet \simeq \int^{\mathcal{C}^{op}} S(\mathcal{C}(-,\bullet), F_-)$$
$$\simeq [\mathcal{C}^{op}, S]_\Lambda(\mathcal{C}(-,\bullet), F_-)$$

$$\begin{tikzcd}
\ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "\mathbb{S}"'] & \Lambda & \Lambda^{op_2} \arrow[l, "(-)^{op}"']
\end{tikzcd}$$

$$\begin{tikzcd}
\Lambda \arrow[r, "\simeq"] \arrow[rrr, bend right, "\Lambda(\ast_\Lambda, -)"'] & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] \arrow[ll, bend right, "{[-,-]_\Lambda}"', "\simeq"] & \mathrm{CAT}
\end{tikzcd}$$

$[\ast_\Lambda, -] \simeq -$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

CAT_2

BUILDING FROM THE POINT $\ast$

$$[\mathcal{C}^{op} \times_\Lambda \mathcal{C}, \mathbb{S}]_\Lambda \xrightarrow{\int^{\mathcal{C}}} \mathbb{S}$$

$$\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda \arrow[r, "\mathcal{C}(-,-)_\ast \times \mathrm{Id}"] \arrow[d, "\mathrm{Id}_\ast"'] \arrow[dr, phantom, "\overset{\text{YONEDA}}{\simeq}"] & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda(-,-)}"] \\
\mathbb{S} &
\end{tikzcd}$$

$$Fx \simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,x), F)$$

$$\ast_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

$$\boxed{\begin{aligned} F_\bullet &\simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,\bullet), F_-) \\ &\simeq [\mathcal{C}^{op}, \mathbb{S}]_\Lambda(\mathcal{C}(-,\bullet), F_-) \end{aligned}}$$

$$\begin{tikzcd}
\ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "\mathbb{S}"'] & \Lambda & \Lambda^{op_2} \arrow[l, "(_)^{op}"']
\end{tikzcd}$$

$$\begin{tikzcd}
 & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[dr, "\Lambda(-,-)"] \arrow[dll, "{[-,-]_\Lambda}", "\simeq"'] & \\
\Lambda \arrow[ur, "\simeq"] \arrow[rrr, "{\Lambda(\ast_\Lambda, -)}"'] & & & \mathrm{CAT}
\end{tikzcd}$$

$[\ast_\Lambda, -] \simeq -$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

CAT_2

BUILDING FROM THE POINT $\ast$

$$[\mathcal{C}^{op} \times_\Lambda \mathcal{C}, \mathbb{S}]_\Lambda \xrightarrow{\int^{\mathcal{C}}} \mathbb{S}$$

$$\mathcal{C} \xhookrightarrow{\mathcal{C}(-,-)_\ast} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

$$\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda \arrow[r, "\mathcal{C}(-,-)_\ast \times \mathrm{Id}"] \arrow[d, "\mathrm{Id}_\ast"'] \arrow[dr, phantom, "\substack{\text{YONEDA} \\ \simeq}"] & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda(-,-)}"] \\
\mathbb{S} &
\end{tikzcd}$$

$$F x \simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,x), F)$$

$$\ast_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

$$\boxed{\begin{aligned} F_\bullet &\simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,\bullet), F_-) \\ &\simeq [\mathcal{C}^{op}, \mathbb{S}]_\Lambda(\mathcal{C}(-,\bullet), F_-) \end{aligned}}$$

$$\mathrm{CAT}_2$$

$$\begin{tikzcd}
 & \Lambda^{op_2} \arrow[d, "(_)^{op}"] & & & \\
\ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "\mathbb{S}"'] & \Lambda \arrow[r, "\simeq"] \arrow[rrr, bend right, "\Lambda(\ast_\Lambda, -)"'] & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] \arrow[ll, bend right, "{[-,-]_\Lambda}"'] & \mathrm{CAT}
\end{tikzcd}$$

$$[\ast_\Lambda, -] \simeq - \qquad \Lambda(\ast_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$$

BUILDING FROM THE POINT $\ast$

$$[\mathcal{C}^{op} \times_\Lambda \mathcal{C}, \mathbb{S}]_\Lambda \xrightarrow{\int^{\mathcal{C}}} \mathbb{S}$$

$$\mathcal{C} \xhookrightarrow{\mathcal{C}(-,-)_\ast} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

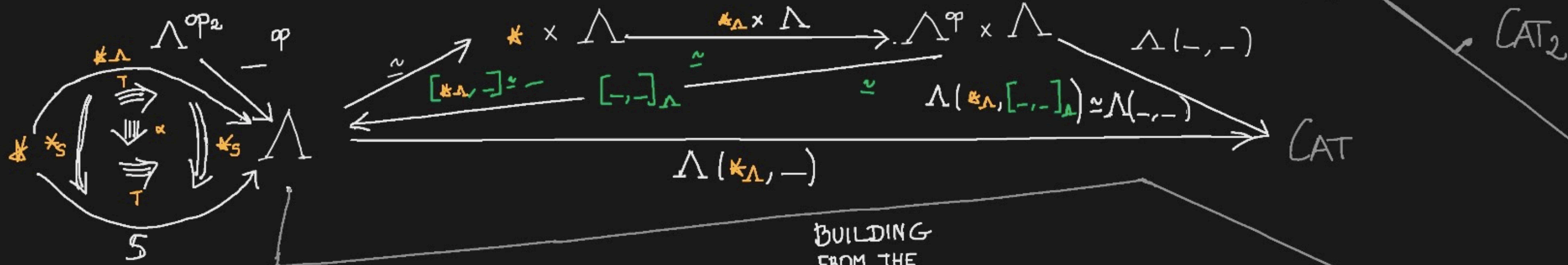
$$\boxed{\mathcal{C}(-,-) \xrightarrow{\simeq} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda(\mathcal{C}(-,-)_\ast, \mathcal{C}(-,-)_\#)}$$

$$\begin{tikzcd}
\mathcal{C}^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda \arrow[r, "\mathcal{C}(-,-)_\# \times \mathrm{Id}"] \arrow[d, "\mathrm{Id}_\#"'] \arrow[dr, phantom, "\text{YONEDA} \simeq"] & {[\mathcal{C}^{op}, \mathbb{S}]_\Lambda^{op} \times_\Lambda [\mathcal{C}^{op}, \mathbb{S}]_\Lambda} \arrow[dl, "{[\mathcal{C}^{op}, \mathbb{S}]_\Lambda(-,-)}"] \\
\mathbb{S} &
\end{tikzcd}$$

$$\ast_\Lambda \xrightarrow{F} [\mathcal{C}^{op}, \mathbb{S}]_\Lambda$$

$$\boxed{\begin{aligned} F_\bullet &\simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,\bullet), F_-) \\ &\simeq [\mathcal{C}^{op}, \mathbb{S}]_\Lambda(\mathcal{C}(-,\bullet), F_-) \end{aligned}}$$

$$Fx \simeq \int^{\mathcal{C}^{op}} \mathbb{S}(\mathcal{C}(-,x), F)$$



BUILDING
FROM THE
POINT $*$

$*_\Lambda$

$\mathcal{CAT}_2$

$$\begin{tikzcd} \ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "S"'] \arrow[r, phantom, "\Downarrow\alpha"] & \Lambda \arrow[r, "\cong"] \arrow[rrr, bend right, "\Lambda(\ast_\Lambda, -)"'] & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] & \mathcal{CAT} \end{tikzcd}$$

Λ^{op_2}, $-^{op}$, $\ast_S$, T

$[\ast_\Lambda, -] \cong -$

$[-,-]_\Lambda$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \cong \Lambda(-,-)$

BUILDING FROM THE POINT $\ast$

$$\ast_\Lambda \xrightarrow{\ast_S} S$$

ACCESSING THE INTERNAL CONTENT OF THE POINT

CAT_2

$$\begin{tikzcd}
 & \Lambda^{op_2} \arrow[dr, "(-)^{op}"] & & \\
\ast \arrow[ur, "\ast_\Lambda"] \arrow[dr, "\ast_S"'] \arrow[r, phantom, "\overset{T}{\Rightarrow}\ \Downarrow\alpha\ \overset{T}{\Rightarrow}"] & & \Lambda \arrow[r, shift left, "\cong"] \arrow[rrr, bend right, "{\Lambda(\ast_\Lambda, -)}"'] & \ast \times \Lambda \arrow[l, shift left, "{[\ast_\Lambda, -] \cong -}"] \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "{\Lambda(-,-)}"] \arrow[ll, bend left, "{[-,-]_\Lambda}", "\cong"'] & CAT \\
 & S \arrow[ur, "\ast_S"'] & & & &
\end{tikzcd}$$

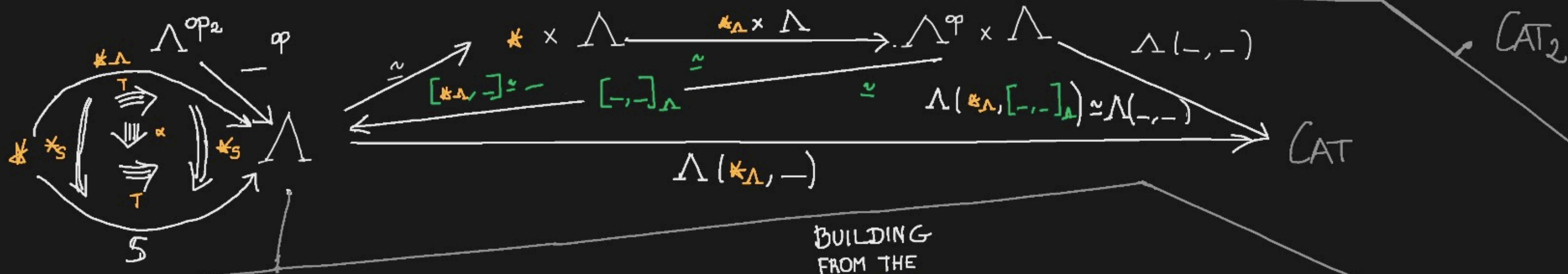
$\cong$

$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \cong \Lambda(-,-)$

BUILDING FROM THE POINT $\ast$

$$\begin{tikzcd}
\ast_\Lambda^{op} \times_\Lambda \ast_\Lambda \arrow[rr, "{\ast_\Lambda(-,-)}"] & & S \\
 & \ast_\Lambda \arrow[ur, "\ast_S"'] &
\end{tikzcd}$$

ACCESSING THE INTERNAL CONTENT OF THE POINT



BUILDING FROM THE POINT $*$

$$\begin{tikzcd}
*_\Lambda^{op} \times_\Lambda *_\Lambda \arrow[rr, "*_\Lambda(-,-)"] \arrow[dr] & \arrow[d, phantom, "\simeq"] & S \\
& *_\Lambda \arrow[ur, "*_S"'] &
\end{tikzcd}$$

ACCESSING THE INTERNAL CONTENT OF THE POINT

CAT_2

$$\begin{tikzcd}
 & \Lambda^{op_2} \arrow[dr, "(-)^{op}"] & \\
* \arrow[ur, "*_\Lambda"] \arrow[rr, bend right, "S"'] \arrow[r, phantom, "\Downarrow\alpha"] & & \Lambda \arrow[r, "\simeq"] \arrow[rrr, bend right, "{\Lambda(*_\Lambda, -)}"'] & * \times \Lambda \arrow[r, "*_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "{\Lambda(-,-)}"] \arrow[ll, bend right, "{[-,-]_\Lambda}"'] & CAT
\end{tikzcd}$$

$*_S$ $\quad$ T $\quad$ $*_S$ $\quad$ T

$[*_\Lambda, -] \simeq -$ $\qquad$ $\Lambda(*_\Lambda, [-,-]_\Lambda) \simeq \Lambda(-,-)$

BUILDING FROM THE POINT $*$

$$\begin{tikzcd}
*_\Lambda^{op} \times_\Lambda *_\Lambda \arrow[rr, "{*_\Lambda(-,-)}"] \arrow[dr] & \simeq & S \\
 & *_\Lambda \arrow[ur, "*_S"'] \arrow[loop left, "*_S"] &
\end{tikzcd}$$

ACCESSING THE INTERNAL CONTENT OF THE POINT

$\mathcal{CAT}_2$

$$
\begin{tikzcd}
\Lambda^{op_2} \arrow[dr, "(-)^{op}"] & & & & \\
\ast \arrow[r, bend left, "\ast_\Lambda"] \arrow[r, bend right, "S"'] \arrow[r, phantom, "\Downarrow\alpha"] & \Lambda \arrow[r, "\cong"] \arrow[rrr, bend right, "\Lambda(\ast_\Lambda, -)"'] & \ast \times \Lambda \arrow[r, "\ast_\Lambda \times \Lambda"] & \Lambda^{op} \times \Lambda \arrow[r, "\Lambda(-,-)"] & \mathcal{CAT}
\end{tikzcd}
$$

$\ast_S$, $\ast_S$, T, T

$[\ast_\Lambda, -] \cong -$

$[-,-]_\Lambda$

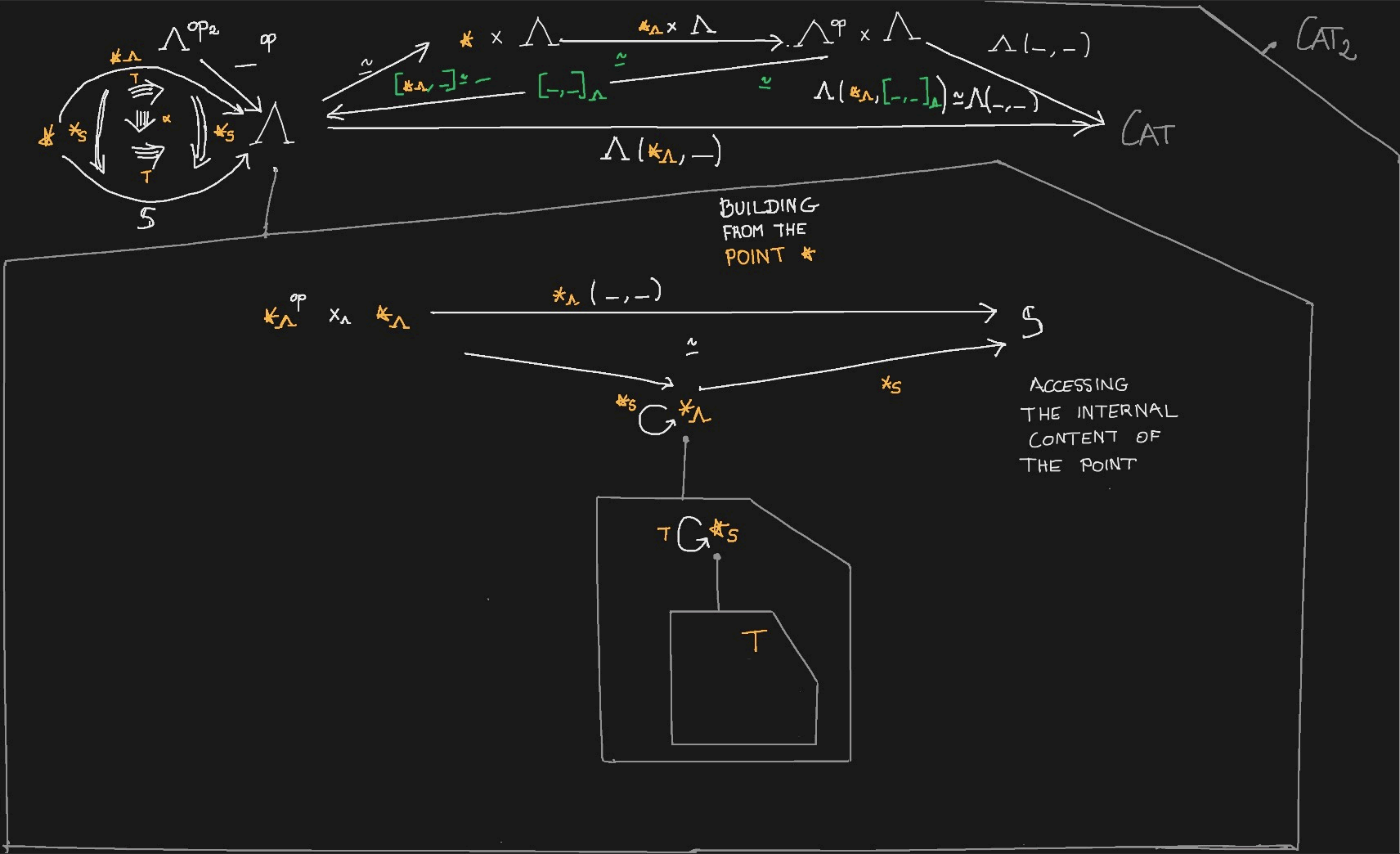
$\Lambda(\ast_\Lambda, [-,-]_\Lambda) \cong \Lambda(-,-)$

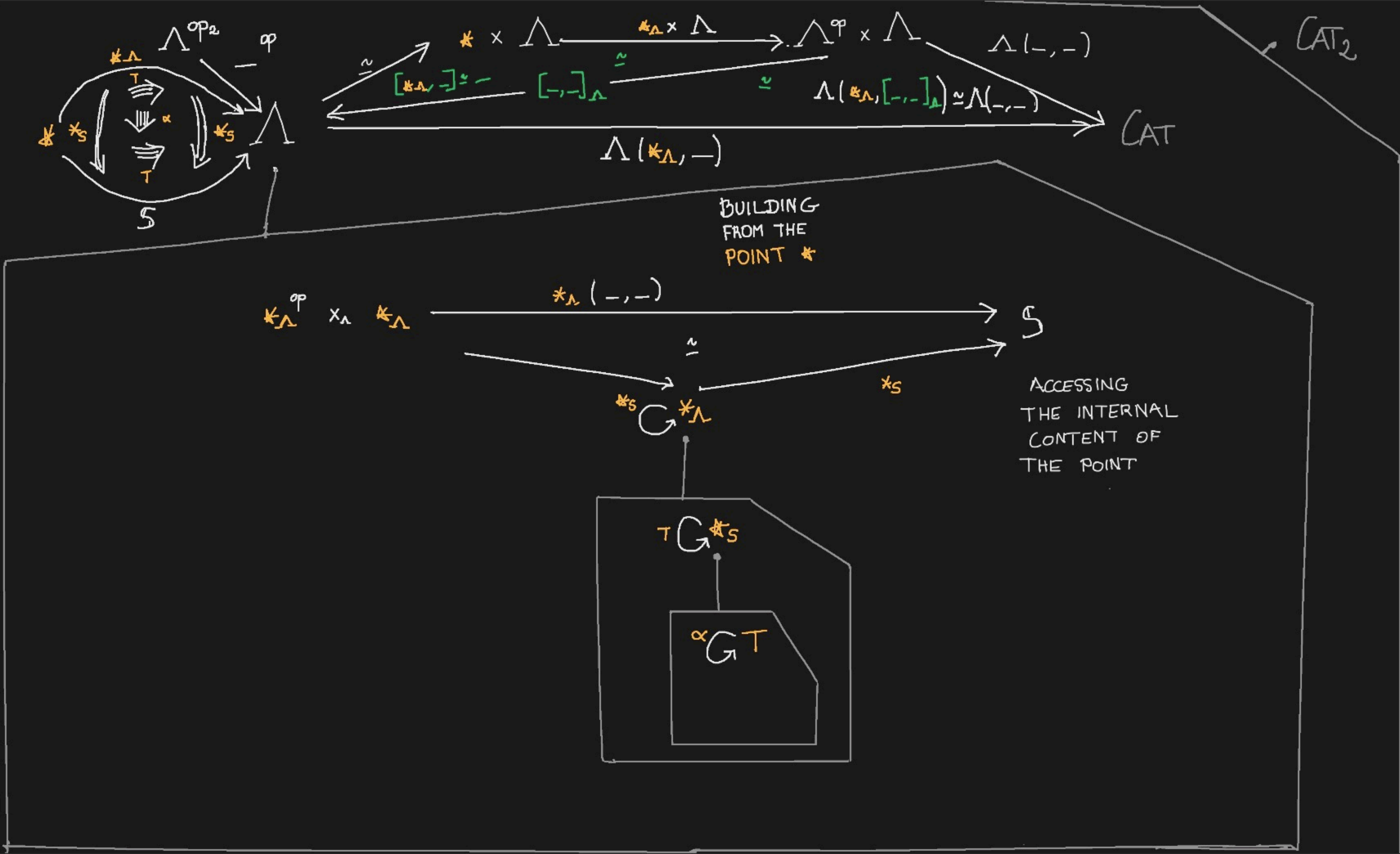
BUILDING
FROM THE
POINT $\ast$

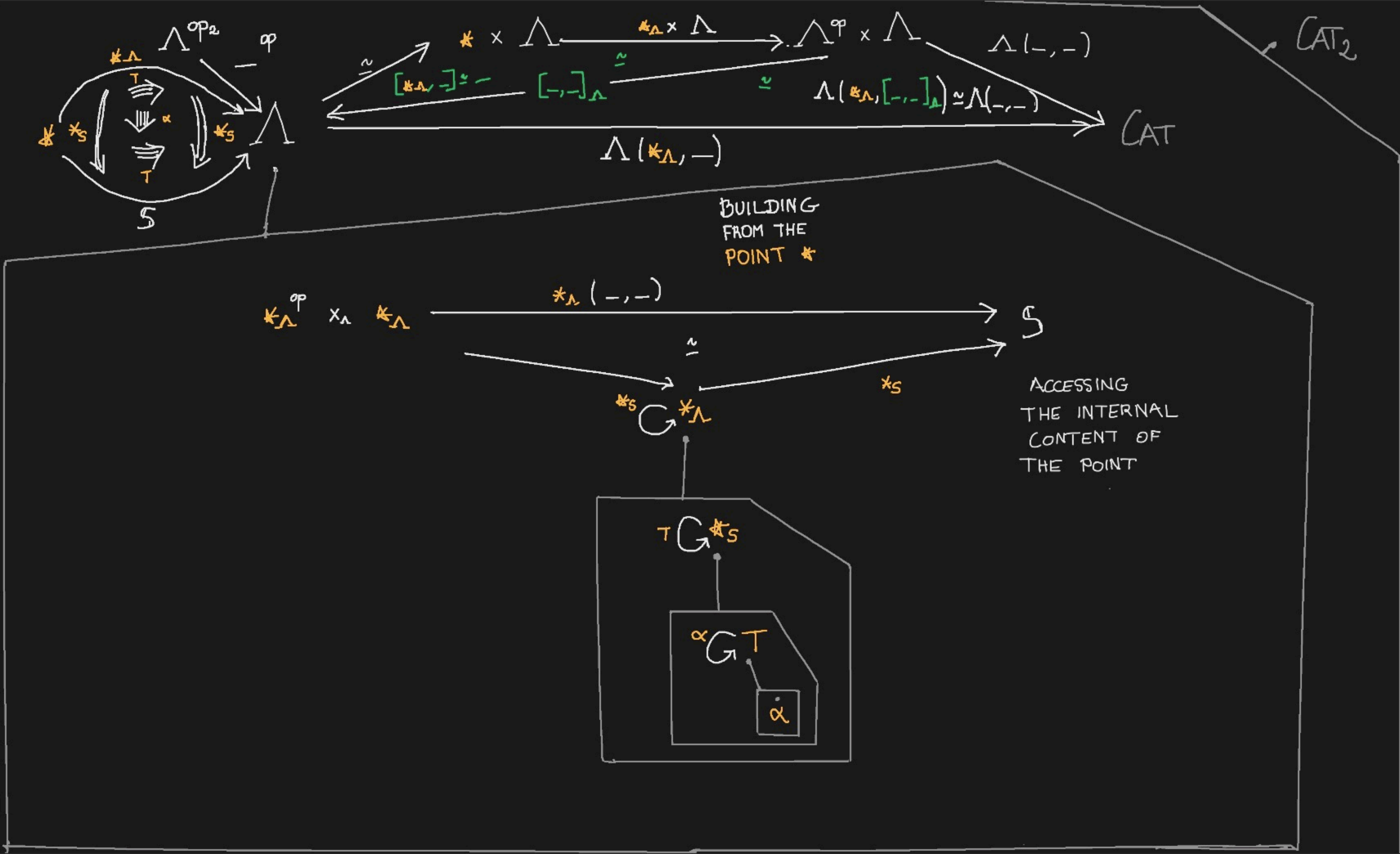
$$
\begin{tikzcd}
\ast_\Lambda^{op} \times_\Lambda \ast_\Lambda \arrow[rr, "\ast_\Lambda(-,-)"] \arrow[dr] & \cong & S \\
& \ast_\Lambda \arrow[loop left, "\ast_S"] \arrow[ur, "\ast_S"'] &
\end{tikzcd}
$$

ACCESSING
THE INTERNAL
CONTENT OF
THE POINT

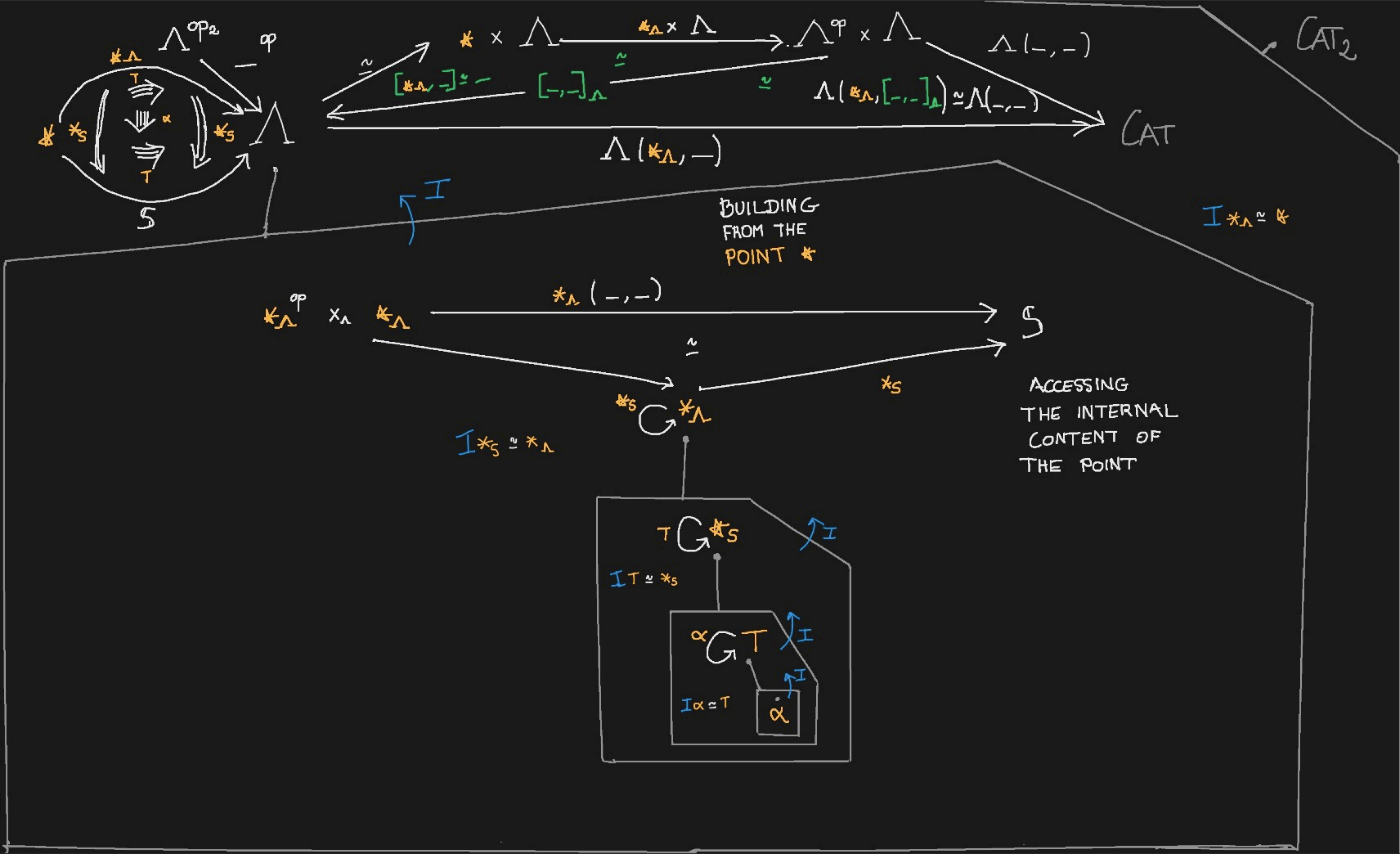
$\ast_S$


CAT₂
CAT
BUILDING FROM THE POINT *
ACCESSING THE INTERNAL CONTENT OF THE POINT
Λ(−,−)
Λ(*Λ,−)
Λ(*Λ,[−,−]Λ) ≃ Λ(−,−)
[*Λ,−] ≃ −
[−,−]Λ
*Λ(−,−)
S
T


CAT₂
CAT
Λ(−,−)
Λ(*Λ,−)
Λ(*Λ,[−,−]Λ) ≃ Λ(−,−)
[*Λ,−] ≃ −
[−,−]Λ
*Λ × Λ
BUILDING FROM THE POINT *
*Λ(−,−)
S
*s
ACCESSING THE INTERNAL CONTENT OF THE POINT



CAT₂
CAT
BUILDING FROM THE POINT *
ACCESSING THE INTERNAL CONTENT OF THE POINT


CAT₂
CAT
Λ(−,−)
Λ(∗Λ,−)
Λ(∗Λ,[−,−]Λ) ≃ Λ(−,−)
[∗Λ,−] ≃ −
[−,−]Λ
BUILDING FROM THE POINT ∗
I∗Λ ≃ ∗
∗Λ(−,−)
I∗S ≃ ∗Λ
IT ≃ ∗S
Iα ≃ T
ACCESSING THE INTERNAL CONTENT OF THE POINT

RADICAL INTERNALISM

PHILOSOPHICAL STANCE

ANY STRUCTURE IS INTERNAL TO ANOTHER

— EVEN WHEN LEFT UNSPOKEN

RETHINKING

- THE DEFINITION OF « DEFINITION »
- THE MEANING OF « MEANING »
- THE FOUNDATIONS OF « THE FOUNDATIONS »