Categories for industrial planning

David Kruml, Jan Paseka

Masaryk University

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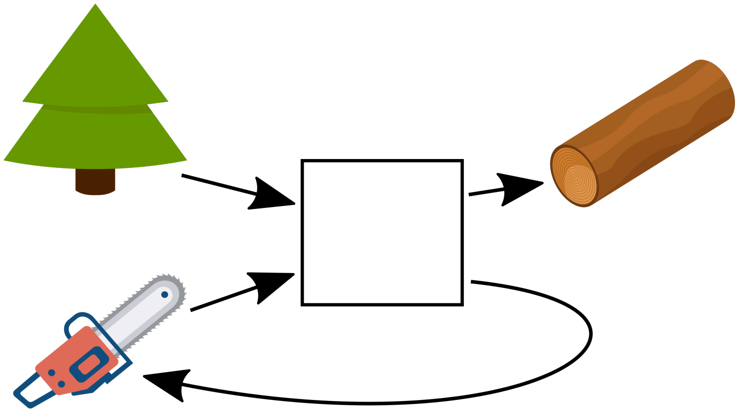
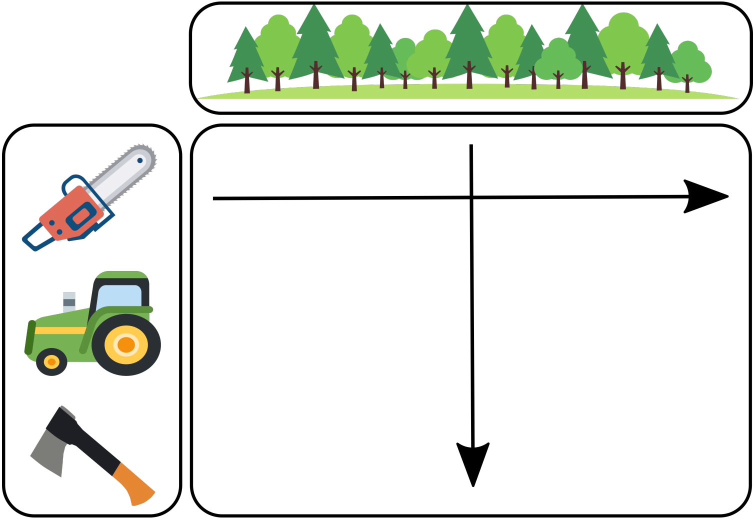
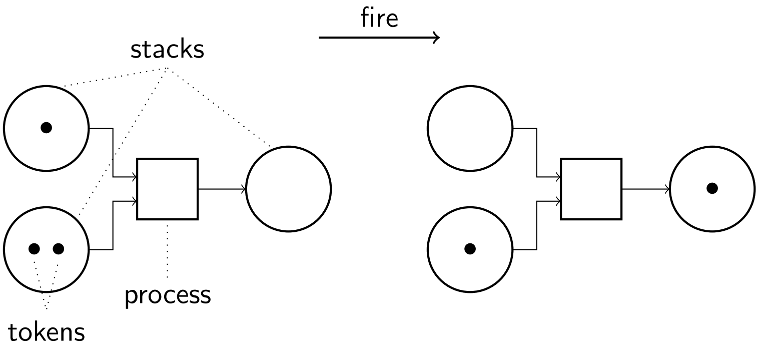
An example of process



Space of events and resource paths

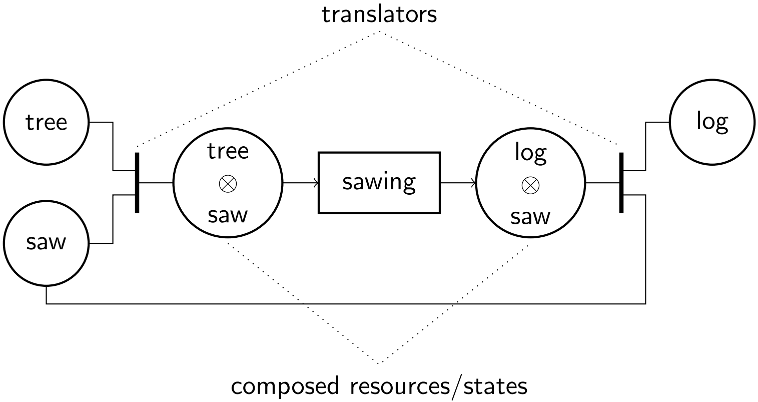


Petri nets



“Tensored” Petri nets

Strongly inspired by the Oxford school (Abramsky, Coecke, et al.) for quantum protocols.



Categorical formalization

- category theory — language, the way of thinking,
- **Proc** — dagger compact "corpus" category of all resources/states (objects) and processes (morphisms)
- I — bounded finite poset,
- *receipt* $R : I \to \mathbf{Proc}$,
- *schedule* (*Gantt diagram*) $S : I \to \mathbb{R}$ (time)
- *plan* = receipt + schedule

“Good” plan

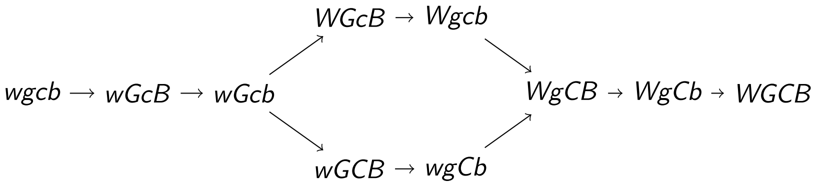
- We focus on resource inventories (MRP, MRP II, ERP): stacks must not underflow neither overflow.
- *Defects* (errors, collisions) are penalized $\Rightarrow$ multi criteria decision, objective function, we can optimize the plan.
- In practice, we prefer “soft constrains” than “hard constrains” — risky strategies could be more profitable (money save most of defects).

Benefits of categorical modeling

- All resources (material, machines, people, energy, externalities, ...) are "emancipated" and modeled the same way. (However, the economists should calculate all the weights for defects.)
- Two types of aggregation:
  - "categorical" — $\circ, \otimes$ (breakdown structures),
  - "instances $\to$ class" — functors (sharing of processes and subreceipts).
- The "logic" of **Proc** seems to be classical (cf. with linear logic of quantum protocols) and probably will be expressed by means of relations ($\Rightarrow$ allegories).
- Indices and orderings on summands, evaluation and comparison of plans $\Rightarrow$ 2-categories?

The wolf, goat, and cabbage problem

- Elementary resources: wolf, goat, cabbage, boat (with the farmer).
- Each item is in one of two states:
  - w, g, c, b — start bank,
  - W, G, C, B — final bank.
- Composed states:
  - $wgcb, wGcB, WgCb, \dots, WGCB$ — acceptable,
  - $wgCB, WgcB, WGcb, \dots$ — forbidden.
- Elementary process (operation): $wgcb \to wGcB$,
- Dagger: $(wgcb \to wGcB)^\dagger = wGcB \to wgcb$.
- Two optimal solutions:



Personalized views $\Rightarrow$ partitions on states/morphisms

- **Farmer's view:** 4 actions: "take a goat" $= \{wgcb \to wGcB, Wgcb \to WGcB, wGcB \to wgcb, \dots\}$, "take a wolf", "take cabbage", "manipulation cruise".
- **Wolf's view:** W/w, "alone with a goat".
- **Optimization view:**
  - forbidden states: penalty -100,
  - acceptable non-terminal states: penalty -1,
  - terminal state $WGCB$: penalty 0.
- Views could be expressed as functors.
- Now the problem is ready to be encoded to constraint programming language and solved with a computer (e. g. MiniZinc solver).

Other questions

- In manufacturing, many resources are "indistinguishable" and processes are repetitive $\Rightarrow$ high level of aggregation, powers of morphisms, etc. Multilevel planning $\Rightarrow$ "higher order regularity" of the flow code.
- Randomness = lack of knowledge, result of aggregation.
- The real planning problems are hard. We optimize by *simulated annealing*.
- In practice, departments of the company can "compete" (e. g. farmer vs. wolf) $\Rightarrow$ different weights, different evaluation of the flow, game theory.
- AI sometimes succeeds in encoding the task but it is still bad in optimization.

Thank you for your attention!