Level é

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Overview

1. Being and Becoming
2. Levels and dimension
3. Two kinds of ‘non-standard’ dimensions
4. The main result

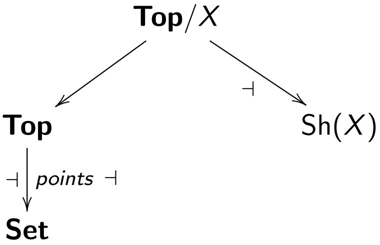
A “positive mathematical program”

Lawvere, F. William
Some thoughts on the future of category theory.
Category theory, Proc. Int. Conf., Como/Italy 1990, LNM 1488, 1-13 (1991).

Being and Becoming

Categories ‘of spaces’ and Generalized-locales

The category Top vs categories of sheaves



for each X in **Top**.

The ‘gros’ Zariski topos vs the ‘petit’ Zariski toposes

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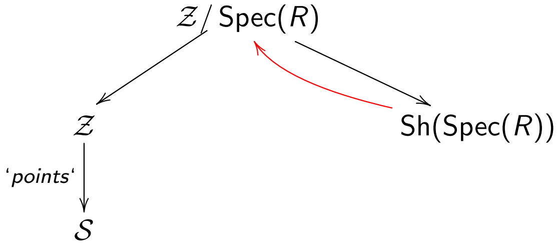

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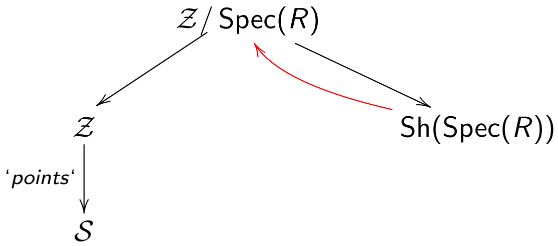

The composite geometric morphism Sh(Spec(R)) $\to \mathcal{Z}$ 'is' the local ring in Sh(Spec(R)) representing R (as the algebra of sections of a sheaf of local rings).

"the important structure sheaf which recalls for the little category the big environment in which it was born"

"A general category of Being, particular categories of Becoming"

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A distinction:

Being vs Becoming
Categories ‘of spaces’ vs Generalized locales

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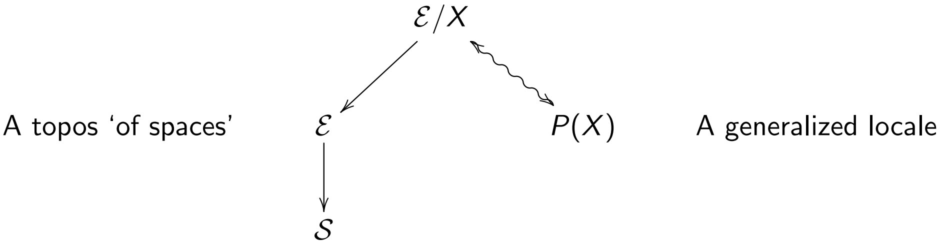

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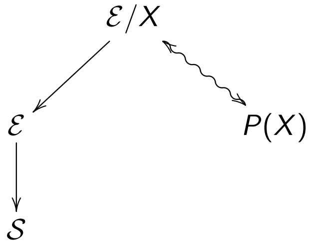

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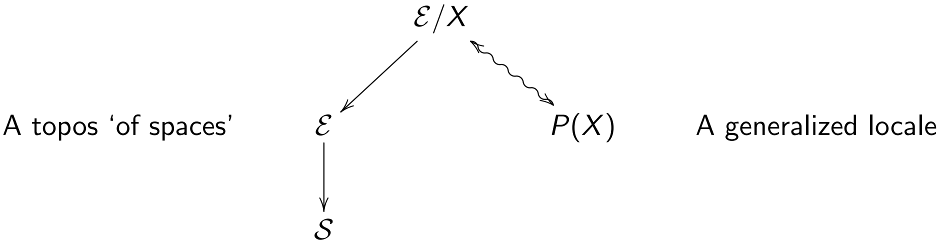

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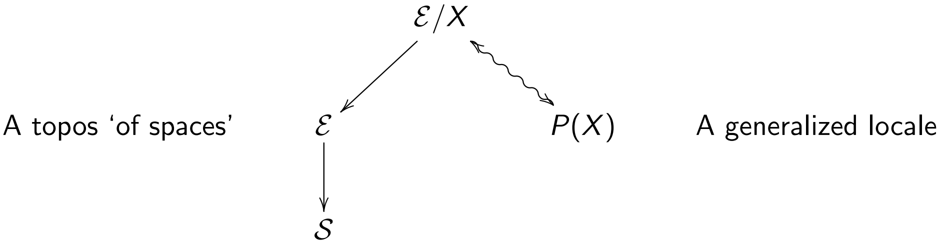

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“Thus one conjectures that $dimX$ only depends on the category $P(X)$ of particular Becoming associated to X”

Toposes ‘of spaces’ vs Generalized locales

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- $\widehat{\Delta}$.
- The Topological topos.
- The Bornological topos.
- The Recursive topos
- The ‘gros’ Zariski topos.
- Models of SDG
- Any pre-cohesive topos.
- ...

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Definition

A topos is locally decidable if...

Back to the motivation

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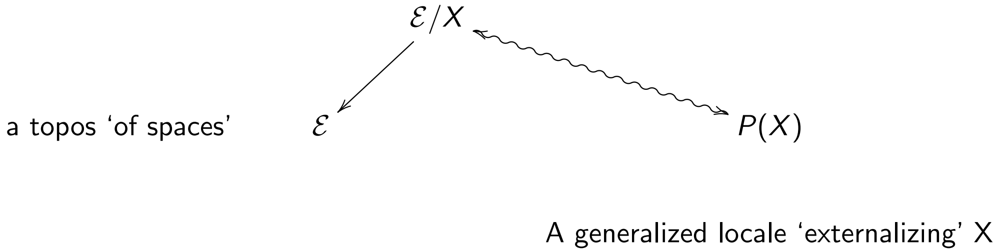

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Dimension Theory

Levels

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A level of a topos $\mathcal{E}$ is an essential subtopos of $l : \mathcal{L} \to \mathcal{E}$. In other words

$$\begin{array}{c} \mathcal{E} \\ l_! \Big\uparrow \;\dashv\; \Big\downarrow l^* \;\dashv\; \Big\uparrow l_* \\ \mathcal{L} \end{array}$$

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We say that $\dim X \leq l$ if the l-skeleton of X is an iso.

So $l_! : \mathcal{L} \to \mathcal{E}$ is the full subcategory of those X s.t. $\dim X \leq l$.

"The basic idea is simply to identify dimensions with levels and then try to determine what the general dimensions are in particular examples. [...]

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8. The Zariski topos? Other toposes in AG, SDG, Rig Geometry?

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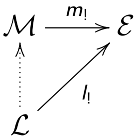

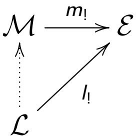

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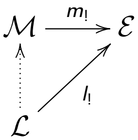

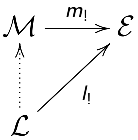

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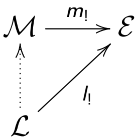

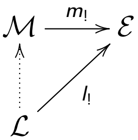

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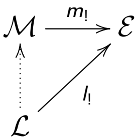

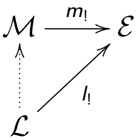

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For instance, the Aufhebung (of a dimension).

Two kinds of ‘non-standard’ dimensions

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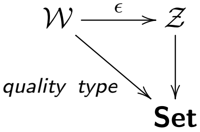
Example

The Zariski topos $\mathcal{Z} \to$ **Set** for $\mathbb{C}$ has a level ϵ and it coincides with Weil topos.



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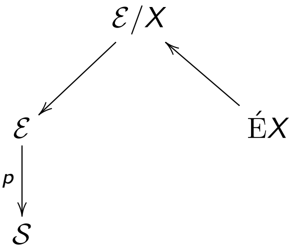

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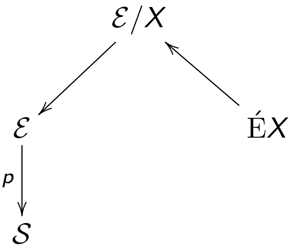

An étendue for each X

“in a site-invariant manner”

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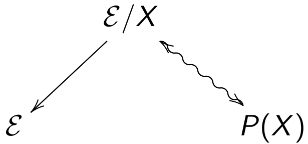
Example

The classifier of non-trivial Boolean algebras, $\widehat{\Delta}$, $\widehat{\Delta_1}$, etc.

The main result

Back to the motivation

a topos ‘of spaces’



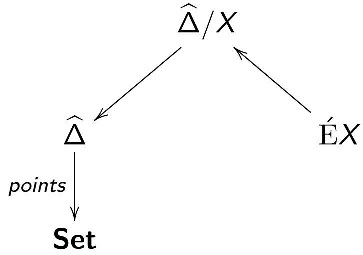
Back to the motivation

$$\begin{array}{ccccc} & & \mathcal{E}/X & & \\ & \swarrow & & \leftrightsquigarrow & \\ \text{a topos `of spaces'} \quad \mathcal{E} & & & & P(X) \end{array}$$

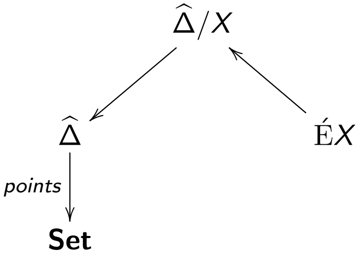
"one conjectures that dim X only depends on the category $P(X)$ of particular Becoming associated to X [...].

In other words, if we have an equivalence of categories $P(X) \equiv P(Y)$, then X, Y should belong to the same class of UIO levels within the category of Being in which they are objects."

The case of simplicial sets



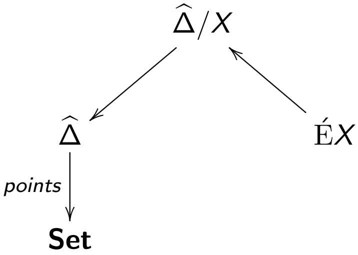
The case of simplicial sets



Theorem

For every strongly regular X, Y *in* $\widehat{\Delta}$,

The case of simplicial sets



Theorem

For every strongly regular X, Y *in* $\widehat{\Delta}$, *if* $\text{É}X \cong \text{É}Y$ *then* $\dim X = \dim Y$.

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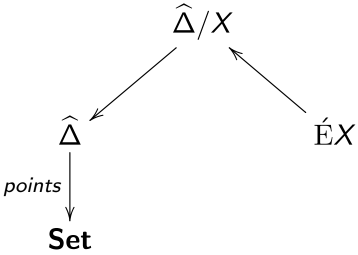

Theorem

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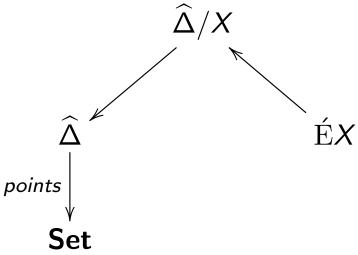
The same result works for many other presheaf toposes.

The case of simplicial sets



Theorem

For every strongly regular X, Y in $\widehat{\Delta}$, if $\acute{\mathrm{E}}X \cong \acute{\mathrm{E}}Y$ then $\dim X = \dim Y$.

The same result works for many other presheaf toposes.

The $\acute{\mathrm{E}}X$ are not always localic.

Example: a non-localic category of Becoming

For the reflexive graph Y

the site for $\acute{\mathrm{E}}Y$ looks as follows

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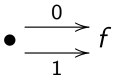
the site for $\text{É}Y$ looks as follows



and is obviously not a poset.

The resulting 'petit' $\text{É}Y$ is the (non-localic) topos of non-reflexive graphs.

Sketch of the proof

Define

$$
\begin{aligned}
\mathrm{IBD}_{-\infty} &:= \bot \\
\mathrm{IBD}_{0} &:= (\forall x : \Omega)(x \vee (x \Rightarrow \mathrm{IBD}_{-\infty})) = (\forall x : \Omega)(x \vee \neg x) \\
\mathrm{IBD}_{n+1} &:= (\forall x : \Omega)(x \vee (x \Rightarrow \mathrm{IBD}_{n}))
\end{aligned}
$$

for every $n \in \mathbb{N}$.

Sketch of the proof

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for every $n \in \mathbb{N}$.

Theorem

For any X in $\widehat{\Delta}$ and $d \in \mathbb{N} + \{-\infty\}$,

$$\text{É}X \text{ satisfies } \mathrm{IBD}_d \quad \text{iff} \quad \dim X \leq d.$$

The ‘petit’ toposes in detail (minimal objects)

Minimal objects

Fix a small category $\mathcal{C}$ with split-epic/mono factorizations.

Lemma

For any object C in $\mathcal{C}$ the following are equivalent:

1. *Every split-epic $C \to D$ is an iso.*
2. *Every $C \to D$ is monic.*

If the above conditions hold then we say that C is minimal.

Definition

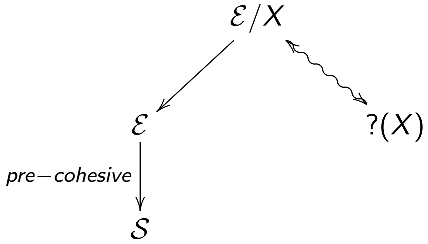
A presheaf X on $\mathcal{C}$ is strongly regular if, for every monic map m in $\widehat{\mathcal{C}}/X$ with minimal codomain, the domain of m is also minimal.

Conclusion

"In other words, if we have an equivalence of categories $P(X) \simeq P(Y)$, then X, Y should belong to the same class of UIO levels within the category of Being in which they are objects.

Suitable hypotheses to make this conjecture true should begin to clarify the relationships between the two suggested philosophical guides."

F. W. Lawvere, *Some thoughts on the future of ct*. LNM 1488.



Thank you for your attention.

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