Unified approach to pointfree T_0-extensions

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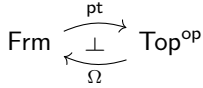
Recall:

The adjunction between frames and spaces:



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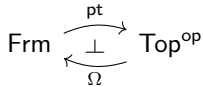

Also note:

- Adjunction is idempotent.
- Fixpoints are sober spaces/spatial frames.

Motivation

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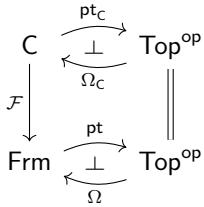
- Relationship with canonical extensions.

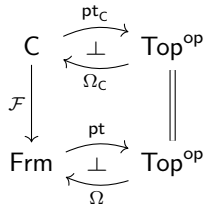
Examples

- Strictly zero-dimensional biframes (Graham, 2018).
- McKinsey-Tarski algebras (Bezhanishvili, Raviprakash, 2023).
- Raney extensions (Suarez, 2025).

The abstraction



The abstraction



The data of $\mathcal{F}\colon \mathsf{C} \to \mathsf{Frm}$ and $\mathsf{pt}_\mathcal{C} \dashv \Omega_\mathsf{C}$ define a *pointfree T_0-extension of the category of frames* if

- the above defines a lax map of adjunctions (right adjoint direction commutes),
- $\mathcal{F}$ is essentially surjective and faithful,
- fixpoints of $\mathsf{pt}_\mathsf{C} \dashv \Omega_\mathsf{C}$ are T_0-spaces.

Some results

Idea: study the posets $\mathcal{F}^{-1}(L)$.

- $\mathcal{F}$ is seldom a fibration.
- Spectrum of initial objects is sober.
- Spectrum of terminal objects is T_D (with extra condition).
- Conjecture: Raney extensions are terminal among such extensions.