Immersions, Submersions, Local Diffeomorphisms, and Relative Cotangent Complexes in Tangent Categories[1]

Geoff Vooys[2] (He/Him)
Based on Joint Work with JS Lemay[3]

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[1]Preprint available at arXiv:2506.07874
[2]University of Calgary
[3]Macquarie University

The Culprits

Figure: JS Lemay (no long hair) and me (long hair) making the Sydney Opera House look good.

The Punchline and History

The Really, Really Ambitious Goal

We will explain how to fill out the following table:

Tan Cat	T-Immers.	T-Unram.	T-Submers.	T-Étale
$\mathbf{SMan}$				
$\mathbf{CAlg}_R$ $R \in \mathbf{CRig}_0$				
$\mathbf{CAlg}_R^{\mathrm{op}}$ $R \in \mathbf{CRig}_0$				
$\mathbf{Sch}_{/S}$ $S \in \mathbf{Sch}_0$				
$\mathscr{C}$ category with finite biproducts				

The Punchline and History

The More Realistic Goal

We will explain how for any reasonable[a] map $f : X \to Y$ in a tangent category, there is a relative cotangent sequence

$$X \longrightarrow T_{X/Y} \xrightarrow{\nu_f} TX \xrightarrow{\theta_f} f^*(TY)$$

in $\mathbf{DBun}(X)$.

[a]This means p-carrable and 0-carrable, which are both defined later.

Outline

The Plan

1. Tangent Categories: What are They?
2. Carrability
3. Immersions and Unramified Maps
4. Submersions and Split Submersions
5. Étale Maps

Tangent Categories

The Core Ideas

- Tangent categories were originally discovered by Rosický in [Rosický1984] and later rediscovered by Cockett and Cruttwell in [CockettCruttwell2014].

Tangent Categories

The Core Ideas

- Tangent categories were originally discovered by Rosický in [Rosický1984] and later rediscovered by Cockett and Cruttwell in [CockettCruttwell2014].
- Tangent categories form a semantic/structural perspective for studying differential geometry by using the structural relations between smooth real manifolds and their tangent bundles.

Tangent Categories

The Core Ideas

- Tangent categories were originally discovered by Rosický in [Rosický1984] and later rediscovered by Cockett and Cruttwell in [CockettCruttwell2014].

- Developing tangent category theory can give new ways to study geometric concepts like vector bundles, connections, smoothness (work in progress), etc. from purely a structural perspective.

Tangent Categories

Definition (cf. [1, Definition 2.3])

A tangent structure $\mathbb{T}$ on a category $\mathscr{C}$ consists of the following data:

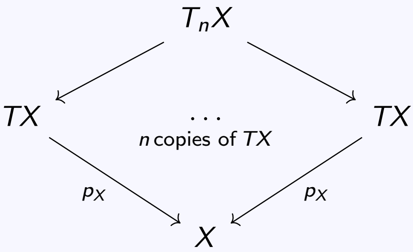
- A functor $T : \mathscr{C} \to \mathscr{C}$ called the tangent functor.

Tangent Categories

Definition (cf. [1, Definition 2.3])

A tangent structure $\mathbb{T}$ on a category $\mathscr{C}$ consists of the following data:

- A natural transformation $p : T \Rightarrow \mathrm{id}_{\mathscr{C}}$ called the bundle projection such that for every object X in $\mathscr{C}$ and for every $n \in \mathbb{N}$ the wide pullback $T_n X$



  exists and is preserved by T^m for all $m \in \mathbb{N}$.

Tangent Categories

Definition (cf. [1, Definition 2.3])

A tangent structure $\mathbb{T}$ on a category $\mathscr{C}$ consists of the following data:

- Natural transformations $0 : \mathrm{id}_{\mathscr{C}} \Rightarrow T$ and $\mathrm{add} : T_2 \Rightarrow T$ called the zero and addition of the tangent structure which make the triple, for all $X \in \mathscr{C}_0$,

$$\left(\begin{array}{c} TX \\ \downarrow p_X \\ X \end{array}, \begin{array}{c} X \\ \downarrow 0_X \\ TX \end{array}, \begin{array}{c} T_2X \\ \downarrow \mathrm{add}_X \\ TX \end{array} \right)$$

  into a commutative monoid in the slice category $\mathscr{C}_{/X}$.

Tangent Categories

Definition (cf. [1, Definition 2.3])

A tangent structure $\mathbb{T}$ on a category $\mathscr{C}$ consists of the following data:

- Natural transformations $\ell : T \Rightarrow T^2$ (called the vertical lift) and $c : T^2 \Rightarrow T^2$ (called the canonical flip) which satisfy certain relations (omitted here).

Tangent Categories

Definition (cf. [1, Definition 2.3])

A tangent category is a pair $(\mathscr{C}, \mathbb{T})$ where $\mathscr{C}$ is a category and $\mathbb{T}$ is a tangent structure on $\mathscr{C}$.

Tangent Categories

Remark

The vertical lift ℓ encodes the local linearity of derivatives while the canonical flip c encodes the symmetry of mixed order partial derivatives

$$\frac{\partial^2 f}{\partial x\, \partial y} = \frac{\partial^2 f}{\partial y\, \partial x}$$

for smooth morphisms f and smooth atlas transition maps.

Example (Examples of Tangent Categories)

Tangent Categories

Example (Examples of Tangent Categories)

Let $\mathscr{C} = \textbf{SMan}$ denote the category of smooth real manifolds[a]. Then $\mathscr{C}$ is a tangent category with:

- Tangent functor given by taking tangent bundles and their derivatives

$$TX := \coprod_{x \in X} T_x X, \qquad Tf := \coprod_{x \in X} D[f](x).$$

- Projection $p_X : TX \to X$ given by bundle projection $(x, \vec{v}) \mapsto x$.
- Zero section $0_X : X \to TX$ given by $x \mapsto (x, \vec{0})$.
- The addition $\operatorname{add} : T_2 X \to TX$ is defined by

$$\left((x, \vec{v}), (x, \vec{w})\right) \mapsto \left(x, \vec{v} + \vec{w}\right).$$

[a] For this talk a smooth manifold is paracompact second countable.

Tangent Categories

Example (Examples of Tangent Categories)

Let $\mathscr{C} = \mathbf{CAlg}_R$ denote the category of commutative R-algebras for a commutative rig[a]. Then $\mathscr{C}$ is a tangent category with:

- Tangent functor $TA := A[\varepsilon] = \{a + b\varepsilon \mid a, b \in A, \varepsilon^2 = 0\}$ the rig of dual numbers with coefficients in A.
- Projection $p_A : TA \to A$ given by $a + b\varepsilon \mapsto a$.
- Zero section $0_A : A \to TA$ given by $a \mapsto a + 0\varepsilon$.
- Addition add : $T_2A \to TA$ is given by

$$a + b\varepsilon_0 + c\varepsilon_1 \mapsto a + (b + c)\varepsilon.$$

[a] A rig, also known as a semiring, is a ring without negatives.

Tangent Categories

Example (Examples of Tangent Categories)

Let S be a base scheme. Then the category $\mathbf{Sch}_{/S}$ is a tangent category but somewhat technical to describe! Its tangent functor is:

- Tangent functor

$$T_{X/S} := \underline{\mathrm{Spec}}_X(\underline{\mathrm{Sym}}_{\mathcal{O}_X}(\Omega^1_{X/S}))$$

  for $\Omega^1_{X/S}$ the sheaf of relative Kähler differentials of X over S, $\underline{\mathrm{Spec}}_X$ the relative spectrum functor, and $\underline{\mathrm{Sym}}_{\mathcal{O}_X}$ the sheafy symmetric algebra functor

$$\mathcal{O}_X : \mathbf{QCoh}(X) \to \mathbf{QCoh}\left(X, \mathbf{CAlg}_{\mathcal{O}_X}\right).$$

Tangent Categories

Example (Examples of Tangent Categories)

The category $\mathscr{C} = \mathbf{CMon}$ of commutative monoids and commutative monoid morphisms is a tangent category with:

- Tangent functor $TM := M \oplus M$ and $Tf := f \oplus f$.
- Projection $\pi_0 : M \oplus M \to M$ given by direct sum projection and zero section given by direct sum inclusion $\iota_0 : M \to M \oplus M$.
- Addition: Defined by the map $\mathrm{add} : M \oplus (M \oplus M) \to M \oplus M$ given as

$$(a, m, n) \mapsto (a, m + n).$$

Tangent Categories

A Fun and Remarkable Fact

Given any[a] tangent category $(\mathscr{C}, \mathbb{T})$ the tangent functor $T : \mathscr{C} \to \mathscr{C}$ is both limit reflecting and colimit reflecting; cf. [LemayVooys25].

[a] Robin Cockett likes to say that "any" is his favourite tangent category.

Carrability

Pullbacks?

It is a well-known problem that **SMan** is not finitely complete and so we cannot expect our tangent categories to have pullbacks save for the ones we require in the definition. As such, we need to *ask* for special situations in which some specified pullbacks exist so that we can recapture horizontal bundles and the cotangent sequence.

Carrability

Pullbacks?

It is a well-known problem that **SMan** is not finitely complete and so we cannot expect our tangent categories to have pullbacks save for the ones we require in the definition. As such, we need to *ask* for special situations in which some specified pullbacks exist so that we can recapture horizontal bundles and the cotangent sequence.
Explicitly, the cases we need to study are when maps admit either pullbacks against the bundle projection p or when their differential admits a pullback against the zero section 0.

Carrability

Definition

We say that a morphism $f : X \to Y$ in a tangent category $\mathscr{C}$ is *p-carrable*[a] if the pullback square

$$\begin{array}{ccc} f^*(TY) & \xrightarrow{\pi_1} & TY \\ {\scriptstyle \pi_0}\downarrow & & \downarrow{\scriptstyle p_Y} \\ X & \xrightarrow[f]{} & Y \end{array}$$

exists in $\mathscr{C}$ and is preserved by T^m for all $m \in \mathbb{N}$.

[a] The word "carrable" is borrowed from the Grothendieck school of algebraic geometry as a word meaning "squarable."

Carrability

Definition

We say that a morphism $f : X \to Y$ in a tangent category $\mathscr{C}$ is *p*-carrable[a] if the pullback square

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exists in $\mathscr{C}$ and is preserved by T^m for all $m \in \mathbb{N}$.
In such a case we call $f^*(TY)$ the horizontal bundle of f and the comparison map $\theta_f : TX \to f^*(TY)$ the horizontal descent of f.

[a] The word "carrable" is borrowed from the Grothendieck school of algebraic geometry as a word meaning "squarable."

Carrability

Zero-Carrable Maps

We say that a morphism $f : X \to Y$ in a tangent category $\mathscr{C}$ is 0-carrable if the pullback square

$$\begin{array}{ccc} T_{X/Y} & \xrightarrow{\mathrm{pr}_1} & Y \\ {\scriptstyle \mathrm{pr}_0}\downarrow & & \downarrow{\scriptstyle 0_Y} \\ TX & \xrightarrow[Tf]{} & TY \end{array}$$

exists in $\mathscr{C}$ and is preserved by T^m for all $m \in \mathbb{N}$.

Carrability

Zero-Carrable Maps

We say that a morphism $f : X \to Y$ in a tangent category $\mathscr{C}$ is 0-carrable if the pullback square

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exists in $\mathscr{C}$ and is preserved by T^m for all $m \in \mathbb{N}$.
In such a case, we call the object $T_{X/Y}$ the vertical bundle of X. This object was first studied by Rosický in his original paper on tangent categories; cf. [Rosický1984, Pages 5 – 6].

Carrability

Examples of Horizontal Bundles

Tangent Category $f : X \to Y$	Horizontal Bundle $f^*(TY) \cong X \times_Y TY$
SMan	$\coprod_{x \in X} T_{f(x)} Y$

Carrability

Examples of Horizontal Bundles

Tangent Category $f : X \to Y$	Horizontal Bundle $f^*(TY) \cong X \times_Y TY$
SMan	$\coprod_{x \in X} T_{f(x)} Y$
$\mathbf{CAlg}_R$	$\{(a, y) \mid a \in X, y \in Y\}$ $(a, y)(c, z) := (ac, f(a)z + yf(c))$

Carrability

Examples of Horizontal Bundles

Tangent Category $f : X \to Y$	Horizontal Bundle $f^*(TY) \cong X \times_Y TY$
SMan	$\coprod_{x \in X} T_{f(x)} Y$
$\mathbf{CAlg}_R$	$X \ltimes Y := \{(a, y) \mid a \in X, y \in Y\}$ $(a, y)(c, z) := (ac, f(a)z + yf(c))$
$\mathbf{CAlg}_R^{\mathrm{op}}$	$\mathrm{Sym}_X(\Omega_{Y/R} \otimes_Y X)$

Carrability

Examples of Horizontal Bundles

Tangent Category $f : X \to Y$	Horizontal Bundle $f^*(TY) \cong X \times_Y TY$
SMan	$\coprod_{x \in X} T_{f(x)} Y$
$\mathbf{CAlg}_R$	$\{(a, y) \mid a \in X, y \in Y\}$ $(a, y)(c, z) := (ac, f(a)z + yf(c))$
$\mathbf{CAlg}_R^{\text{op}}$	$\mathrm{Sym}_X(\Omega_{Y/R} \otimes_Y X)$
$\mathscr{C}$ with finite biproducts	$X \oplus Y$

Carrability

Examples of Vertical Bundles

Tangent Category $f : X \to Y$	Vertical Bundle $T_{X/Y}$
SMan	$\{(x, \overrightarrow{v}) \in TX \mid D[f](x)\overrightarrow{v} = \overrightarrow{0}\}$

Carrability

Examples of Vertical Bundles

Tangent Category $f : X \to Y$	Vertical Bundle $T_{X/Y}$
SMan	$\{(x, \overrightarrow{v}) \in TX \mid D[f](x)\overrightarrow{v} = \overrightarrow{0}\}$
$\mathbf{CAlg}_R$	$\{(a, y) \mid f(y) = 0\}$

Examples of Vertical Bundles

Tangent Category $f : X \to Y$	Vertical Bundle $T_{X/Y}$
SMan	$\{(x, \overrightarrow{v}) \in TX \mid D[f](x)\overrightarrow{v} = \overrightarrow{0}\}$
$\mathbf{CAlg}_R$	$\{(a, y) \mid f(y) = 0\}$
$\mathbf{CAlg}_R^{op}$	$\mathrm{Sym}_X(\Omega_{X/Y})$

Carrability

Examples of Vertical Bundles

Tangent Category $f : X \to Y$	Vertical Bundle $T_{X/Y}$
SMan	$\{(x, \vec{v}) \in TX \mid D[f](x)\vec{v} = \vec{0}\}$
$\mathbf{CAlg}_R$	$\{(a, y) \mid f(y) = 0\}$
$\mathbf{CAlg}_R^{op}$	$\mathrm{Sym}_X(\Omega_{X/Y})$
$\mathscr{C}$ with finite biproducts	$X \oplus \mathrm{Ker}(f)$ if $\mathrm{Ker}(f)$ exists

The Relative Cotangent Sequence

Differential Bundles

In tangent categories we have notions of what are called differential bundles by [CockettCruttwell2018].
These are the tangent-categorical analogue of both vector bundles (in the smooth manifold case) *and* of quasi-coherent sheaves (in the algebraic-geometric case) and are similarly indispensable.
We will not fully define them here, but will note their basic structure.

The Relative Cotangent Sequence

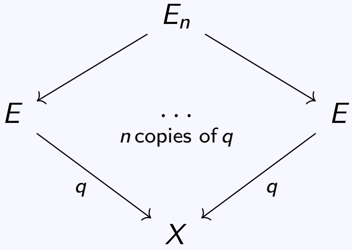
Differential Bundles, Abbreviated

Let $\mathscr{C}$ be a tangent category and let X be an object of $\mathscr{C}$. A differential bundle q over X consists of the following data:

The Relative Cotangent Sequence

Differential Bundles, Abbreviated

A morphism $q : E \to X$ in $\mathscr{C}$ such that for all $n \in \mathbb{N}$ the wide pullback



exists and is preserved by T^m for all $m \in \mathbb{N}$.

The Relative Cotangent Sequence

Differential Bundles, Abbreviated

There is a specified section $\zeta : X \to E$ of q and a morphism $\sigma : E_2 \to E$ which make

$$\left(\begin{array}{c} E \\ \downarrow q \\ X \end{array}, \begin{array}{c} X \\ \downarrow \zeta \\ E \end{array}, \begin{array}{c} E_2 \\ \downarrow \sigma \\ E \end{array} \right)$$

into a commutative monoid in $\mathscr{C}_{/X}$.

The Relative Cotangent Sequence

Differential Bundles, Abbreviated

There is a specified lift $\lambda : E \to TE$ such that maps $q, \zeta, \sigma, \lambda$ satisfy various coherences which encode the local linearity required of a vector bundle/module.

We will often abuse notation and write E (the domain of the projection) in place of specifying the full differential bundle

$$\mathsf{q} = \left(\begin{array}{c} E \\ \downarrow q \\ X \end{array}, \begin{array}{c} X \\ \downarrow \zeta \\ E \end{array}, \begin{array}{c} E_2 \\ \downarrow \sigma \\ E \end{array}, \begin{array}{c} E \\ \downarrow \lambda \\ TE \end{array} \right)$$

The Relative Cotangent Sequence

Definition

A linear morphism of differential bundles over X is a morphism $f : E \to F$ for which the diagrams

$$
\begin{array}{ccc}
E & \xrightarrow{f} & F \\
& \searrow \quad \swarrow & \\
& X &
\end{array}
\qquad
\begin{array}{ccc}
E & \xrightarrow{f} & F \\
{\scriptstyle \lambda_E}\downarrow & & \downarrow{\scriptstyle \lambda_F} \\
TE & \xrightarrow[Tf]{} & TF
\end{array}
$$

commute.

The Relative Cotangent Sequence

Definition

A linear morphism of differential bundles over X is a morphism $f : E \to F$ for which the diagrams

$$\begin{array}{ccc} E & \xrightarrow{\quad f \quad} & F \\ & \searrow \quad \swarrow & \\ & X & \end{array} \qquad \begin{array}{ccc} E & \xrightarrow{f} & F \\ {\scriptstyle \lambda_E}\downarrow & & \downarrow{\scriptstyle \lambda_F} \\ TE & \xrightarrow[Tf]{} & TF \end{array}$$

commute.

We write $\mathbf{DBun}(X)$ for the category of differential bundles over a specified $X \in \mathscr{C}_0$ and the linear maps between them.

The Relative Cotangent Sequence

Some Examples of Differential Bundles

Tangent Category with object X	Equivalent Formulation of $\mathbf{DBun}(X)$
$\mathbf{SMan}_{\text{Par.comp.}}^{\text{H.dorff}}$	$\mathbf{Vec}(X)$
$\mathbf{CAlg}_R$	$X\text{-}\mathbf{Mod}$
$\mathbf{CAlg}_R^{\text{op}}$	$X\text{-}\mathbf{Mod}^{\text{op}}$
$\mathbf{Sch}_{/S}$	$\mathbf{QCoh}(X)^{\text{op}}$

Note that the equivalence of $\mathbf{DBun}(X) \simeq \mathbf{Vec}(X)$ for paracompact Hausdorff smooth manifolds is the main theorem of [MacAdam2021] while the remaining characterizations are the results of [CruttwellLemay2023].

The Relative Cotangent Sequence

Two Lone Facts

An important result of [Lucyshyn-Wright2018] is that the pullback of differential bundles, $E \times_X F$, in $\mathbf{DBun}(X)$ is a biproduct whenever it exists!

In particular, the differential object X is a zero object of $\mathbf{DBun}(X)$ via the bundle structure $X = (\mathrm{id}, \mathrm{id}, \mathrm{id}, 0_X)$.

The Relative Cotangent Sequence

Theorem

Assume that $f : X \to Y$ is a 0-carrable and p-carrable morphism in $\mathscr{C}$. Then the diagram

$$T_{X/Y} \xrightarrow{\mathrm{pr}_0} TX \underset{\underline{0}}{\overset{\theta_f}{\rightrightarrows}} f^*(TY)$$

is an equalizer in $\mathbf{DBun}(X)$. In particular, $T_{X/Y}$ is the kernel of θ_f.

The Relative Cotangent Sequence

Definition

Let $f : X \to Y$ be a p-carrable and 0-carrable morphism in a tangent category. The relative cotangent sequence of f in $\mathbf{DBun}(X)$ is:

$$X \longrightarrow T_{X/Y} \xrightarrow{\mathsf{pr}_0} TX \xrightarrow{\theta_f} f^*(TY)$$

Definition

Let $f : X \to Y$ be a p-carrable and 0-carrable morphism in a tangent category. The relative cotangent sequence of f in $\mathbf{DBun}(X)$ is:

$$X \longrightarrow T_{X/Y} \xrightarrow{\mathrm{pr}_0} TX \xrightarrow{\theta_f} f^*(TY)$$

Remark

We can use this to define submersions in tangent categories as precisely the maps for which the relative cotangent sequence presents θ_f as a cokernel of pr_0:

$$X \longrightarrow T_{X/Y} \xrightarrow{\mathrm{pr}_0} TX \xrightarrow{\theta_f} f^*(TY) \longrightarrow X$$

The "Alas, I am Out of Time" Slide

A List of Definitions

Here is a list of generalizations we can make in tangent categories just by using p-carrability or 0-carrability:

The “Alas, I am Out of Time” Slide

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- T-immersions are maps $f : X \to Y$ for which θ_f is monic.

The "Alas, I am Out of Time" Slide

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Here is a list of generalizations we can make in tangent categories just by using p-carrability or 0-carrability:

- T-immersions are maps $f : X \to Y$ for which θ_f is monic.
- T-submersions are maps $f : X \to Y$ for which θ_f is regular epic.

The "Alas, I am Out of Time" Slide

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- T-immersions are maps $f : X \to Y$ for which θ_f is monic.
- T-submersions are maps $f : X \to Y$ for which θ_f is regular epic.
- Split T-submersions are maps $f : X \to Y$ for which θ_f is a retract.

The "Alas, I am Out of Time" Slide

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- T-immersions are maps $f : X \to Y$ for which θ_f is monic.
- T-submersions are maps $f : X \to Y$ for which θ_f is regular epic.
- Split T-submersions are maps $f : X \to Y$ for which θ_f is a retract.
- T-étale maps are those maps for which θ_f is an isomorphism.

The "Alas, I am Out of Time" Slide

A List of Definitions

Here is a list of generalizations we can make in tangent categories just by using p-carrability or 0-carrability:

- T-immersions are maps $f : X \to Y$ for which θ_f is monic.
- T-submersions are maps $f : X \to Y$ for which θ_f is regular epic.
- Split T-submersions are maps $f : X \to Y$ for which θ_f is a retract.
- T-étale maps are those maps for which θ_f is an isomorphism.
- T-unramified maps are the maps for which $T_{X/Y} \cong X$.

The “Alas, I am Out of Time” Slide

A Remarkable Fact

In full generality, T-immersions are not the same as being T-unramified! In **CMon** (a category with biproducts) the map $\mathrm{sum} : \mathbb{N} \oplus \mathbb{N} \to \mathbb{N}$ is T-unramified but not a T-immersion.

References

Bibliography

- [CockettCruttwell2014] J. R. B. Cockett and G. S. H. Cruttwell, *Differential structure, tangent structure, and SDG*, Appl. Categ. Structures 22 (2014), no. 2, 331 – 417.
- [CockettCruttwell2018] —, *Differential bundles and fibrations for tangent categories*, Cah. Topol. Géom. Différ. Catég. 59 (2018), no. 1, 10-–92.
- [CruttwellLemay2023] G. S. H. Cruttwell and J-S. P. Lemay, *Differential bundles in commutative algebra and algebraic geometry*, Theory Appl. Categ. 39 (2023), Paper No. 36, 1077–1120
- [LemayVooys2025] J.-S. P. Lemay and G. Vooys, *Horizontal Descent, Immersions, Unramified Morphisms, Submersions, Étale Morphisms, and the Relative Cotangent Sequence in Tangnet Categories*. Preprint avialable at arXiv:2506.07874
- [Lucyshyn-Wright2018] R. B. B. Lucyshyn-Wright, *On the geometric notion of connection and its expression in tangent categories*, Theory Appl. Categ. 33 (2018), Paper No. 28, 832-–866.
- [MacAdam2021] B. MacAdam, *Vector bundles and differential bundles in the category of smooth manifolds*, Appl. Categ. Structures 29 (2021), no. 2, 285–310
- [Rosický1984] *Abstract tangent functors*, Diagrammes 12 (1984), JR1—JR11

The Last Slide

The End

Thanks for coming and listening everybody!

Bonus Facts for the Viewers at Home

A Classification of T-Immersions

A morphism $f : X \to Y$ is a T-immersion whenever θ_f is monic.

Category	T-Immersions
SMan	Immersion
$\mathbf{CAlg}_R$	Monic algebra map
$\mathbf{CAlg}_R^{\mathrm{op}}$	$f^{\mathrm{op}} : B \to A$ with $\Omega_{B/A} \cong 0$
$\mathbf{Sch}_{/S}$	$f : X \to Y$ with $\Omega_{X/Y} \cong 0$
$\mathscr{C}$ with biproducts	f monic

Bonus Facts for the Viewers at Home

A Classification of T-Unramified Maps

A morphism $f : X \to Y$ is T-unramified whenever $T_{X/Y} \cong X$ in $\mathbf{DBun}(X)$.

Category	T-Unramified
$\mathbf{SMan}$	Immersion
$\mathbf{CAlg}_R$	Monic algebra map
$\mathbf{CAlg}_R^{\mathrm{op}}$	$f^{\mathrm{op}} : B \to A$ with $\Omega_{B/A} \cong 0$
$\mathbf{Sch}_{/S}$	$f : X \to Y$ with $\Omega_{X/Y} \cong 0$
$\mathscr{C}$ with biproducts	f with trivial kernel

Bonus Facts for the Viewers at Home

A Classification of T-submersions

A morphism $f : X \to Y$ is a T-submersion whenever θ_f is epic.

Category	T-Submersion
SMan	Submersion
$\mathbf{CAlg}_R$	Surjective algebra map
$\mathbf{CAlg}_R^{\mathrm{op}}$	$f^{\mathrm{op}} : B \to A$ with short exact cotangent sequence
$\mathbf{Sch}_{/S}$	$f : X \to Y$ with short exact cotangent sequence
$\mathscr{C}$ with biproducts	f regular epic

Bonus Facts for the Viewers at Home

A Classification of spilt T-submersions

A morphism $f : X \to Y$ is a split T-submersion whenever θ_f is a retract.

Category	T-Submersion
SMan	Submersion with connection
$\mathbf{CAlg}_R$	Surjective algebra map
$\mathbf{CAlg}_R^{\mathrm{op}}$	$f^{\mathrm{op}} : B \to A$ with split short exact cotangent sequence
$\mathbf{Sch}_{/S}$	$f : X \to Y$ with split short exact cotangent sequence
$\mathscr{C}$ with biproducts	f retract

Bonus Facts for the Viewers at Home

A Classification of T-étale Maps

A morphism $f : X \to Y$ is T-étale whenever θ_f is an isomorphism.

Category	T-Étale
SMan	Local diffeomorphism
$\mathbf{CAlg}_R$	Isomorphism
$\mathbf{CAlg}_R^{\text{op}}$	$f^{\text{op}} : B \to A$ with $\Omega_{B/R} \cong \Omega_{A/R} \otimes_A B$ cotangent sequence
$\mathbf{Sch}_{/S}$	$f : X \to Y$ with $\Omega_{X/S} \cong f^* \Omega_{Y/S}$
$\mathscr{C}$ with biproducts	f isomorphism