Quasi-homeomorphisms of topological groupoids

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Topoi with enough points and topological groupoids, arXiv:2408.15848

Quasi-homeomorphisms of spaces

The Grothendieck topos of *sheaves* on a space $\mathrm{Sh}(X)$ is a useful formalism to study algebraic structure defined over a space, e.g. *principal bundles*, *sheaf cohomology*.

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Definition

A continuous map $f\colon X \to Y$ is called a *quasi-homeomorphism* if

$$f^{-1}\colon \mathcal{O}(Y) \to \mathcal{O}(X)$$

is a bijection.

Theorem (Grothendieck [Gr66])

Let X and Y be T_0-spaces, then the following are equivalent:

(i) X and Y have equivalent categories of *sheaves*,

(ii) X and Y admit subspace embeddings

$$X \subseteq W \supseteq Y$$

by quasi-homeomorphisms.

Topoi and groupoids

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In the point-free setting:

Theorem (Moerdijk [Mo88a])

Two *localic* groupoids $\mathbb{X}$, $\mathbb{Y}$ have the same topoi of sheaves if and only if there exist continuous functors

$$\mathbb{X} \leftarrow \mathbb{W} \rightarrow \mathbb{Y}$$

which are *essential equivalences* (internally to locales).

Main result and overview

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that are *quasi-homeomorphisms*.

I. We recall the construction of the *topos of sheaves* on a topological groupoid.

II. We define the class of *logical groupoids*.

III. We identify the class of *quasi-homeomorphisms* of logical groupoids.

Topological groupoids

Definition

A topological groupoid $\mathbb{X} = (X_1 \rightrightarrows X_0)$ consists of a groupoid

$$X_1 \times_{X_0} X_1 \xrightarrow{m} X_1 \overset{\xrightarrow{t}}{\underset{\xrightarrow{s}}{\xleftarrow{e}}} X_0,$$

with $i : X_1 \to X_1$,

where X_0 and X_1 are endowed with topologies making all the above maps continuous.

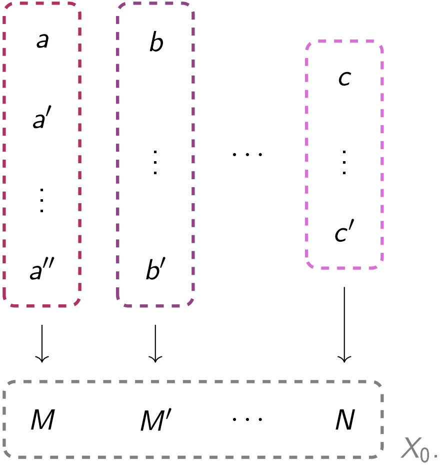
If s (equivalently, t) is open, we say $\mathbb{X}$ is an *open* topological groupoid.

Equivariant sheaves on a groupoid

A *sheaf* on $\mathbb{X}$ consists of:

(i) a local homeomorphism $q\colon Y \to X_0$,

(ii) and a continuous X_1-action $X_1 \times_{X_0} Y \to Y$.



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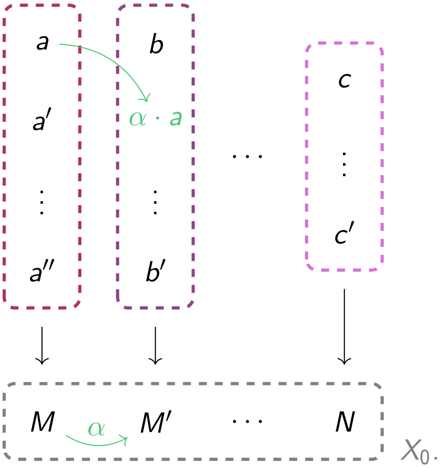

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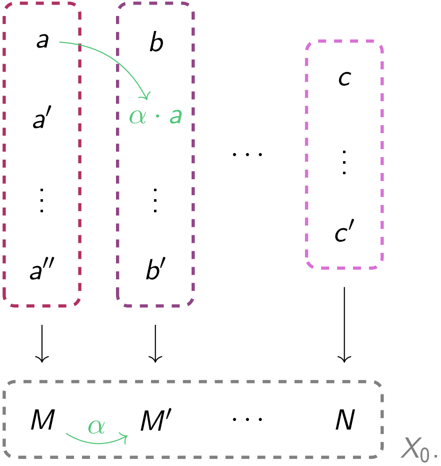

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The category of sheaves and their morphisms define a topos $\mathrm{Sh}(\mathbb{X})$.

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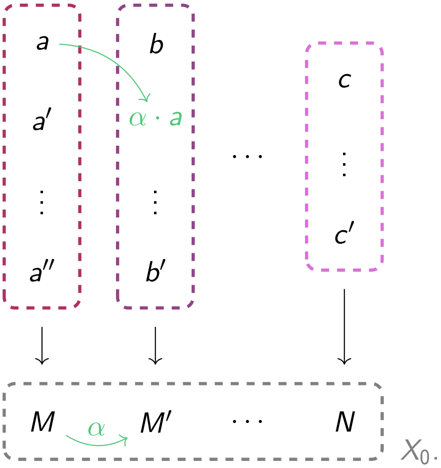

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Example

A space X defines an 'identities only' topological groupoid.

Its topos of sheaves is the usual $\mathrm{Sh}(X)$.

Example

A topological group G is a topological groupoid.

Its sheaves is the topos $\mathbf{B}G$ of continuous actions by G on discrete sets.

Logical groupoids

Proposition

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(ii) The topology τ on G is the coarsest topology determined by the topos $\mathrm{B}G$ –

i.e. if σ is another topology on G for which $\mathrm{B}G^\sigma$ is canonically equivalent to $\mathrm{B}G^\tau$, then

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(iii) There exists a set X such that $G \subseteq \Omega(X)$.

Here, $\Omega(X)$ is endowed with the *pointwise convergence* topology, generated by the subsets

$$\{ \alpha \in \Omega(X) \mid \alpha(x) = y \}$$

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We say that a topological groupoid

$$\mathbb{X} = (X_1 \rightrightarrows X_0)$$

is *logical* if

- $\mathbb{X}$ is an open topological groupoid,
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Proposition

Every topos with enough points is equivalent to the topos of sheaves on a logical groupoid.

Quasi-homeomorphisms of topological groupoids

Let $\mathbb{X}$ be a topological groupoid, and let $\mathbb{Y}, \mathbb{U} \subseteq \mathbb{X}$ be subgroupoids.

Each arrow $\alpha \in X_1$ comes with a left $\mathbb{Y}$-action and a right $\mathbb{U}$-action:

$$(\beta, \alpha) \mapsto \beta \circ \alpha, \ (\alpha, \gamma) \mapsto \alpha \circ \gamma,$$

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The *bi-orbit* for these actions is the set

$${}_{\mathbb{Y}}[\alpha]_{\mathbb{U}} = \{ \beta \circ \alpha \circ \gamma \mid \beta \in Y_1, \gamma \in U_1 \}.$$

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Let $\mathbb{X}$ be a logical groupoid and let $\mathbb{Y} \subseteq \mathbb{X}$ be a subgroupoid.

The inclusion $\mathbb{Y} \subseteq \mathbb{X}$ yields an equivalence

$$\mathsf{Sh}(\mathbb{Y}) \simeq \mathsf{Sh}(\mathbb{X})$$

if and only if, for each open subgroupoid $\mathbb{U} \subseteq \mathbb{X}$, the map

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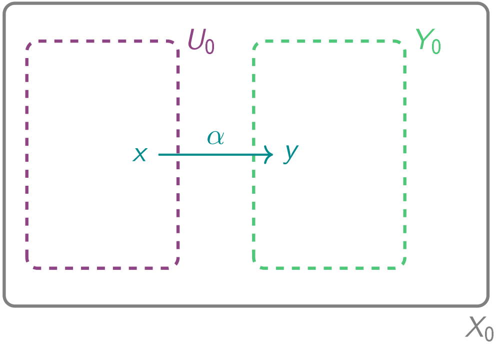

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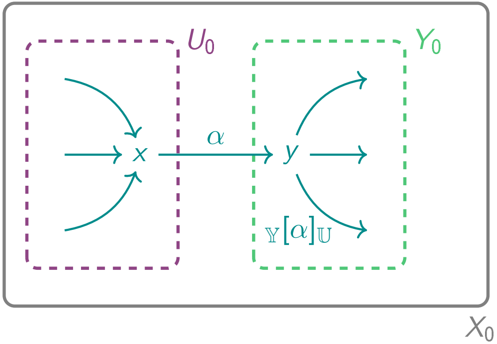

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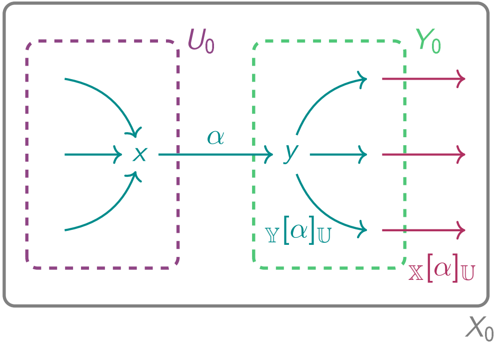

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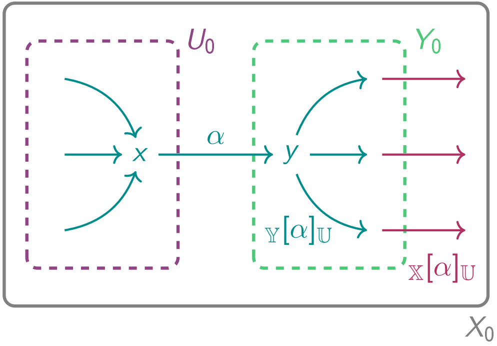

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is a quasi-homeomorphism.

Definition

A subgroupoid inclusion $\mathbb{Y} \subseteq \mathbb{X}$ satisfying the theorem is said to be a *quasi-homeomorphism* of logical groupoids.

Sketch proof

A subgroup $G \subseteq \Omega(X)$ induces a *relational structure* Σ_G on the set X (see Hodges [Ho93]).

If $H \subseteq G \subseteq \Omega(X)$, then $\Sigma_H \supseteq \Sigma_G$ is a *relational expansion* (called a *localic* expansion in Caramello [Ca18]).

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If $\mathbb{X}$ is a logical groupoid and $\mathbb{Y} \subseteq \mathbb{X}$ is a subgroupoid, then the induced morphism

$$\iota \colon \mathrm{Sh}(\mathbb{Y}) \to \mathrm{Sh}(\mathbb{X})$$

is a *localic* geometric morphism.

Thus, ι is an equivalence if and only if it induces an isomorphism on subobjects

$$\mathrm{Sub}_{\mathrm{Sh}(\mathbb{X})}(W) \to \mathrm{Sub}_{\mathrm{Sh}(\mathbb{Y})}(\iota^* W)$$

for each $W \in \mathrm{Sh}(\mathbb{X})$ in a generating set of $\mathrm{Sh}(\mathbb{X})$.

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For an open subgroupoid $\mathbb{U} \subseteq \mathbb{X}$, the space $[U_0 \to X_0]_{\mathbb{U}}$ yields a sheaf on $\mathbb{X}$. These form a generating set for the topos $\mathrm{Sh}(\mathbb{X})$.

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Thus, ι is an equivalence if and only if it induces an isomorphism on subobjects

$$\mathsf{Sub}_{\mathsf{Sh}(\mathbb{X})}([U_0 \to X_0]_{\mathbb{U}}) \to \mathsf{Sub}_{\mathsf{Sh}(\mathbb{Y})}(\iota^*[U_0 \to X_0]_{\mathbb{U}})$$

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Thus, ι is an equivalence if and only if it induces an isomorphism on subobjects

$$\mathcal{O}({}_{\mathbb{X}}[U_0 \to X_0]_{\mathbb{U}}) \cong \mathsf{Sub}_{\mathsf{Sh}(\mathbb{X})}([U_0 \to X_0]_{\mathbb{U}}) \to \mathsf{Sub}_{\mathsf{Sh}(\mathbb{Y})}(\iota^*[U_0 \to X_0]_{\mathbb{U}}) \cong \mathcal{O}({}_{\mathbb{Y}}[U_0 \to Y_0]_{\mathbb{U}})$$

for each open subgroupoid $\mathbb{U} \subseteq \mathbb{X}$.

A subobject of $[U_0 \to X_0]_{\mathbb{U}}$ is an open subspace that is closed under the X_1-action

$$(\beta, [\alpha]_{\mathbb{U}}) \mapsto [\beta \circ \alpha]_{\mathbb{U}},$$

and similarly for $\iota^*[U_0 \to X_0]_{\mathbb{U}} \cong [U_0 \to Y_0]_{\mathbb{U}}$.

Proposition

There are isomorphisms

$$\mathcal{O}({}_{\mathbb{X}}[U_0 \to X_0]_{\mathbb{U}}) \cong \mathsf{Sub}_{\mathsf{Sh}(\mathbb{X})}([U_0 \to X_0]_{\mathbb{U}}),$$
$$\mathcal{O}({}_{\mathbb{Y}}[U_0 \to Y_0]_{\mathbb{U}}) \cong \mathsf{Sub}_{\mathsf{Sh}(\mathbb{Y})}(\iota^*[U_0 \to X_0]_{\mathbb{U}}).$$

Consequences

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that are quasi-homeomorphisms.

(ii) Given two logical groupoids $\mathbb{X}$, $\mathbb{Y}$, every geometric morphism $f\colon \mathrm{Sh}(\mathbb{X}) \to \mathrm{Sh}(\mathbb{Y})$ is induced by a diagram of *continuous functors*

$$\begin{array}{ccc} & & \mathbb{Y} \\ & & \cap\!| \\ \mathbb{X} & \xrightarrow{f} & \mathbb{W} \end{array}$$

where $\mathbb{Y} \subseteq \mathbb{W}$ is a quasi-homeomorphism.

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Thank you for listening