Infinite and Non-Rigid Reconstruction Theory

Or: Reconstruction for lax module monads

Based on joint work with Matti Stroiński: arXiv:2409.00793

2025-07-17

Tony Zorman

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Tannaka duality

$$\mathscr{C} \underset{T}{\circlearrowright}$$

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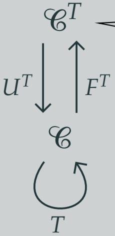
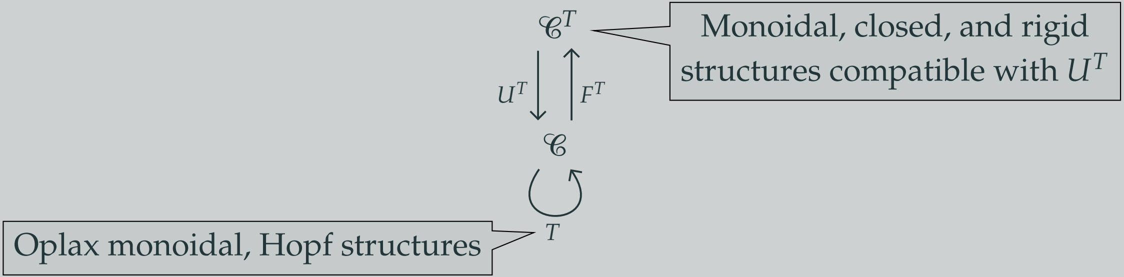
$$
\begin{array}{c}
\mathscr{C}^T \\
U^T \big\downarrow \quad \big\uparrow F^T \\
\mathscr{C} \\
\circlearrowleft \\
T
\end{array}
$$

Tannaka duality



Monoidal, closed, and rigid structures compatible with U^T

Tannaka duality



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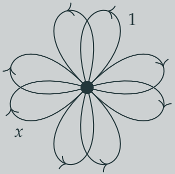
such that for all $x, y, z \in \mathscr{C}$ and $m \in \mathscr{M}$, e.g.,

$$(x \otimes y) \otimes z \cong x \otimes (y \otimes z) \qquad \text{and} \qquad (x \otimes y) \triangleright m \cong x \triangleright (y \triangleright m).$$

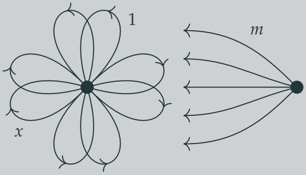
Module categories as deloopings

The **delooping** of a monoidal category is a bicategory with one object.



Module categories as deloopings

The **delooping** of a module category is a bicategory with two objects.



Given a monoidal category $\mathscr{C}$, are all left $\mathscr{C}$-module categories equivalent to the modules of an algebra object in $\mathscr{C}$?

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Proposition (Douglas–Schommer-Pries–Snyder)

In the absence of rigidity, there are finite abelian $\mathscr{C}$-module categories that cannot be realised as the modules of an algebra object in $\mathscr{C}$.

Given a monoidal category $\mathcal{C}$, are all left $\mathcal{C}$-module categories equivalent to the modules of a monad on $\mathcal{C}$?

Lax module functors

A functor $F\colon \mathcal{M} \longrightarrow \mathcal{N}$ is a **lax $\mathcal{C}$-module functor** if there exists an associative and unital natural transformation

$$F_2\colon - \triangleright F(=) \Longrightarrow F(- \triangleright =).$$

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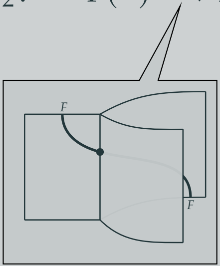

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- **oplax**: $F(- \triangleright =) \Longrightarrow - \triangleright F(=)$;
- **strong**: $- \triangleright F(=) \overset{\sim}{\Longrightarrow} F(- \triangleright =)$.

The Yoneda lemma™

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To Kelly and Beck

Theorem (Kelly)

Given an adjunction $F\colon \mathscr{C} \rightleftarrows \mathscr{D} : U$ between monoidal categories, oplax monoidal structures on F are in bijective correspondence with lax monoidal structures on U.

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Theorem (Beck's monadicity theorem)

An adjunction $F\colon \mathscr{C} \rightleftarrows \mathscr{D} : U$ is monadic if and only if U is conservative, $\mathscr{D}$ has coequalisers of U-split pairs, and U preserves them.

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Theorem (Abelian monadicity)

An adjunction $F\colon \mathcal{C} \rightleftarrows \mathcal{D} : U$ is monadic if U is exact and reflects zero objects.

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Example

- Every object in a rigid monoidal category $\mathscr{C}$ is $\mathscr{C}$-projective.
- Finite $\mathscr{C}$-module categories over finite tensor categories always admit $\mathscr{C}$-projective $\mathscr{C}$-generators.

The Eilenberg–Moore category of a lax $\mathscr{C}$-module monad does *not* carry a canonical $\mathscr{C}$-module structure.

A module structure for the Eilenberg–Moore category

Theorem (Linton, Day, Aguiar–Haim–López Franco, Stroiński–Z)

The Eilenberg–Moore category of any right exact lax $\mathscr{C}$-module monad can be equipped with a canonical $\mathscr{C}$-module structure by means of Linton coequalisers.

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Definition

The **Linton coequaliser** of $x \in \mathscr{C}$ and $(m, \nabla_m) \in \mathcal{M}^T$ is:

$$T(x \triangleright Tm) \underset{\mu_{x \triangleright m} \circ T T_{2;x,m}}{\overset{T(x \triangleright \nabla_m)}{\rightrightarrows}} T(x \triangleright m) \twoheadrightarrow x \blacktriangleright m.$$

The reconstruction result

Theorem (Stroiński–Z)

Let $\mathscr{C}$ be an abelian monoidal category, $\mathscr{M}$ an abelian $\mathscr{C}$-module category, and assume that $\ell \in \mathscr{M}$ is a closed $\mathscr{C}$-projective $\mathscr{C}$-generator.

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Furthermore, there is a bijection

$$\{(\mathscr{M}, \ell) \text{ as above}\} / \mathscr{M} \simeq \mathscr{N} \stackrel{\cong}{\longleftrightarrow} \left\{ \begin{matrix} \text{Right exact lax } \mathscr{C}\text{-module} \\ \text{monads on } \mathscr{C} \end{matrix} \right\} / \; \mathscr{C}^T \simeq \mathscr{C}^S$$

Thanks!

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Reconstruction of module categories in the infinite and non-rigid settings.
arXiv:2409.00793

Hopf trimodules

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Let B be a bialgebra, and define $\mathcal{V} := {}^{B}\mathsf{Vect}$. There is a monoidal equivalence

$$\begin{aligned} {}^{B}_{B}\mathsf{Vect}^{B} &\longrightarrow \mathsf{LexfLax}\mathcal{V}\mathsf{Mod}(\mathcal{V},\mathcal{V}) \\ X &\longmapsto (X \,\square_B\, -, \chi) \end{aligned}$$

between the category of Hopf trimodules, and the category of left exact finitary lax $\mathcal{V}$-module endofunctors on $\mathcal{V}$.

The Yetter–Drinfeld braiding

For all $M, N \in {}^B\mathsf{Vect}$, the arrow

$$\chi_{M,N} \colon M \otimes_{\Bbbk} (X \,\square_B\, N) \longrightarrow X \,\square_B\, (M \otimes_{\Bbbk} N)$$

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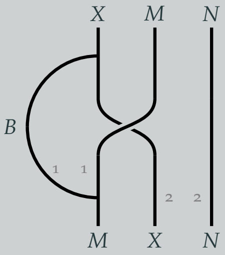

Deducing a theorem for Hopf trimodules

Proposition

Let $\mathscr{C}$ be a left closed monoidal category such that the canonical embedding

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is an equivalence. Then $\mathscr{C}$ is left rigid.

Corollary (Stroiński–Z)

A bialgebra B admits a twisted antipode if and only if the canonical functor $B \otimes_{\Bbbk} -\colon {}^{B}\mathsf{Vect} \longrightarrow {}^{B}_{B}\mathsf{Vect}^{B}$ is an equivalence.

Fusion operators for Hopf monads

Proposition (Stroiński–Z)

The bimonad $T := UF$ of an oplax monoidal adjunction $F\colon \mathscr{C} \rightleftarrows \mathfrak{D} :U$ is canonically an oplax $\mathfrak{D}$-module monad.

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The right fusion operator is the "free part" of the coherence morphism:

$$T_{2;F,\mathrm{Id}} = T_{\mathsf{rf}} : T(T \otimes \mathrm{Id}) \Longrightarrow T \otimes T.$$

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and T_{rf} is an isomorphism if and only if T_2 is.

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