Matter coupling to higher spin supermultiplets

Pavel Kočí

Masaryk University

in collaboration with

Konstantinos Koutrolikos and Rikard von Unge
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Introduction

What are higher spins (HS)?

- elementary particles - unitary irep's of the Poincare group
  $m \neq 0, s = 0$ - Higgs boson
  $m \neq 0, s = 1/2$ - [neutrinos]
  $m = 0, s = 1$ - EM, non-abelian YM
  $m = 0, s = 3/2$ - local SUSY
  $m = 0, s = 2$ - GR
- higher spins - $s > 2$

Motivation:

- QFT in full generality
- string theory - massless modes + tower of masive HS

Our work:

- cubic interactions between a single matter superfield and a tower of HS supermultiplets
  [Buchbinder, Gates, Koutrolikos '17] [Koutrolikos, PK, von Unge '17]

Cubic interactions and matter supermultiplets

Noether's method:

$$\left.\begin{array}{l}\phi \text{ - matter fields} \\ h \text{ - gauge fields}\end{array}\right\} \to \text{ a perturbative way to find interactions}$$

$$S[\phi, h] = S_0[\phi] + S_0[h] + gS_1[\phi, h] + g^2(\dots)\ ,$$
$$\delta\phi = g\delta_1[\phi, \xi] + g^2(\dots)\ ,$$
$$\delta h = \delta_0[\zeta] + g(\dots)\ ,$$

$$g\frac{\delta S_0}{\delta \phi}\delta_1\phi + g\frac{\delta S_1}{\delta h}\delta_0 h = 0\ .$$

Our case:

ϕ - a single matter superfield
h - a tower of HS supermultiplets

4D $N = 1$ superspace: $\{\mathrm{D}, \overline{\mathrm{D}}\} = i\partial,$ $\int d^8z = \int d^4x\ \mathrm{D}^2\overline{\mathrm{D}}^2$

$$S_0^\Phi = \int d^8z\ \Phi\overline{\Phi} \qquad S_0^\Sigma = -\int d^8z\ \Sigma\overline{\Sigma}$$

$$\text{def}: \overline{\mathrm{D}}\Phi = 0 \qquad \Leftarrow \text{duality} \Rightarrow \qquad \text{def}: \overline{\mathrm{D}}^2\Sigma = 0$$

$$\text{eom}: \mathrm{D}^2\Phi = 0 \qquad \text{eom}: \mathrm{D}\Sigma = 0$$

Review of higher spin supermultiplets

- the integer superspin $Y = s$, propagating spins $(s + 1/2, s)$:

$$\delta_0 \Psi_{\alpha(s)\dot\alpha(s-1)} = -\mathrm{D}^2 L_{\alpha(s)\dot\alpha(s-1)} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}\Lambda_{\alpha(s)\dot\alpha(s-2))} \ ,$$
$$\delta_0 V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s} L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\bar{L}_{\alpha(s-1)\dot\alpha(s)} \ ,$$

- the half-integer superspin $Y = s + \frac{1}{2}$, $(s + 1, s + 1/2)$:
  transverse theory:

$$\delta_0 H_{\alpha(s)\dot\alpha(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s} L_{\alpha(s)\dot\alpha(s-1))} \ ,$$
$$\delta_0 \chi_{\alpha(s)\dot\alpha(s-1)} = \bar{\mathrm{D}}^2 L_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot\alpha(s-1)} \ ,$$

  longitudinal theory:

$$\delta_0 H_{\alpha(s)\dot\alpha(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s} L_{\alpha(s)\dot\alpha(s-1))} \ ,$$
$$\delta_0 \chi_{\alpha(s-1)\dot\alpha(s-2)} = \bar{\mathrm{D}}^{\dot\alpha_{s-1}}\mathrm{D}^{\alpha_s} L_{\alpha(s)\dot\alpha(s-1)} + \frac{s-1}{s}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^{\dot\alpha_{s-1}} L_{a(s)\dot\alpha(s-1)}$$
$$+\bar{\mathrm{D}}_{\dot\alpha_{s-2}} J_{\alpha(s-1)\dot\alpha(s-3)} \ .$$

  $\alpha(s)$ - s undotted, symmetric indices: $\alpha_1, \alpha_2, \ldots, \alpha_s$

'Higher spin' transformation for Σ

- the superdiffeomorphism (coupling to SUGRA)

$$\delta\Sigma = \Delta^{\alpha}\mathrm{D}_{\alpha}\Sigma + \Delta^{\dot{\alpha}}\overline{\mathrm{D}}_{\dot{\alpha}}\Sigma + i\Delta^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\Sigma + \Delta\Sigma$$

- the 'HS' generalization of the above

$$\begin{aligned}\delta_1\Sigma = g\sum_{k=0}^{\infty}\Big\{ & A^{\alpha(k+1)\dot{\alpha}(k)}\ \mathrm{D}_{\alpha_{k+1}}\overline{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\overline{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Sigma \\ & +B^{\alpha(k)\dot{\alpha}(k+1)}\ \overline{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}_{\alpha_k}\overline{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\overline{\mathrm{D}}_{\dot{\alpha}_1}\Sigma \\ & +\Gamma^{\alpha(k)\dot{\alpha}(k)}\ \overline{\mathrm{D}}_{\dot{\alpha}_k}\mathrm{D}_{\alpha_k}\ldots\overline{\mathrm{D}}_{\dot{\alpha}_1}\mathrm{D}_{\alpha_1}\Sigma \\ & +\Delta^{\alpha(k)\dot{\alpha}(k)}\ \mathrm{D}_{\alpha_k}\overline{\mathrm{D}}_{\dot{\alpha}_k}\ldots\mathrm{D}_{\alpha_1}\overline{\mathrm{D}}_{\dot{\alpha}_1}\Sigma\Big\}\end{aligned}$$

- δ_1 has to preserve Σ : $\overline{\mathrm{D}}^2\left(\delta_1\Sigma\right) = 0$

$$A_{\alpha(k+1)\dot{\alpha}(k)} = -\overline{\mathrm{D}}^{\dot{\alpha}_{k+1}}\Gamma_{\alpha(k+1)\dot{\alpha}(k+1)} + \tfrac{1}{k+1}\overline{\mathrm{D}}^2\ell_{\alpha(k+1)\dot{\alpha}(k)}\ ,\ k = 0, 1\ldots\ ,$$

$$\Delta_{\alpha(k)\dot{\alpha}(k)} = \overline{\mathrm{D}}^{\dot{\alpha}_{k+1}}B_{\alpha(k)\dot{\alpha}(k+1)} + \tfrac{1}{k!}\overline{\mathrm{D}}_{(\dot{\alpha}_k}\ell_{\alpha(k)\dot{\alpha}(k-1))}\ ,\ k = 1, 2, ..\ ,$$

$$\Gamma + \Delta = \overline{\mathrm{D}}^{\dot{\alpha}}B_{\dot{\alpha}} + \overline{\mathrm{D}}^2\ell\ .$$

- the correct transformation of χ - $\overline{B}_{\alpha(k+1)\dot{\alpha}(k)} = -\overline{\mathrm{D}}^2\ell_{\alpha(k+1)\dot{\alpha}(k)}$
motivating by the superdiffeomorphism - $\Gamma_{\alpha(k)\alpha(k)} = \Delta_{\alpha(k)\dot{\alpha}(k)}\ ,\ k = 1, 2, \ldots\ ,$

Coupling to higher spin supermultiplets

Cubic interactions with higher spins

$$S_1 = \int d^8z \sum_{k=0}^{\infty} \Big\{ \quad H^{\alpha(k+1)\dot\alpha(k+1)} \overbrace{\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)}}^{\text{supercurrent}} + \chi^{\alpha(k+1)\dot\alpha(k)} \overbrace{\mathrm{D}_{\alpha_{k+1}} \mathcal{T}_{\alpha(k)\dot\alpha(k)}}^{\text{supertrace}} + c.c. \Big\}$$

$$- \int d^8z \; V \; \mathcal{J}$$

where

$$\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} = X_{\alpha(k+1)\dot\alpha(k+1)} + \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar{U}_{\alpha(k))\dot\alpha(k+1)} - \frac{1}{(k+1)!}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\mathrm{D}^2 U_{\alpha(k+1)\dot\alpha(k))} \;,$$

$$\mathcal{T}_{\alpha(k)\dot\alpha(k)} = Y_{\alpha(k)\dot\alpha(k)} - \frac{k+2}{k+1} Z_{\alpha(k)\dot\alpha(k)} + \frac{k+2}{k+1}\, \mathrm{D}^{\alpha_{k+1}} U_{\alpha(k+1)\dot\alpha(k)} + \bar{\mathrm{D}}^{\dot\alpha_{k+1}} \bar{U}_{\alpha(k))\dot\alpha(k+1)} \;,$$

$$\mathcal{J} = -\Sigma\bar\Sigma$$

Minimal higher spin supercurrent multiplet

Minimal supercurrent multiplet

$$\mathcal{J}^{min}_{\alpha(k+1)\,\dot\alpha(k+1)},\ \mathcal{T}^{min}_{\alpha(k)\,\dot\alpha(k)} = 0 \rightarrow \text{conformal HS}\ .$$

How to cancel the supertrace?

- $\mathcal{T}_{\alpha(k)\,\dot\alpha(k)}$ is not unique

$$\mathcal{T}_{\alpha(k)\,\dot\alpha(k)} \sim \mathcal{T}_{\alpha(k)\,\dot\alpha(k)} + \mathrm{D}_{(\alpha_k}\mathcal{G}_{\alpha(k-1)\,\dot\alpha(k))} + \overline{\mathrm{D}}^2\left(\mathcal{H}_{\alpha(k)\,\dot\alpha(k)}\right)\ ,$$

- absorb trivial terms by the redefinition $\Sigma = \hat\Sigma + g\ \overline{\mathrm{D}}^{\dot\alpha}\mathcal{Y}_{\dot\alpha}$

$$S_0 = -\int d^8z\ \hat\Sigma\ \bar{\hat\Sigma}\ + g\int d^8z\ \overline{\mathcal{Y}}^{\alpha}\ \mathrm{D}_\alpha\hat\Sigma\ + g\int d^8z\ \mathcal{Y}^{\dot\alpha}\ \overline{\mathrm{D}}_{\dot\alpha}\bar{\hat\Sigma}\ + g^2(\dots)$$

- using the ansatz for $U_{\alpha(k+1)\dot\alpha(k)}$

$$U_{\alpha(k+1)\dot\alpha(k)} = \sum_{p=0}^{k} f_p\ \partial^{(p)}\Sigma\ \partial^{k-p}\lambda\ ,\ \ \Sigma = \overline{\mathrm{D}\lambda}$$

The first three minimal supercurrents:

$$\mathcal{J}^{min}_{\alpha\dot\alpha} = -\frac{i}{3}\ \partial_{\alpha\dot\alpha}\Sigma\ \overline{\Sigma} + \frac{i}{3}\ \Sigma\ \partial_{\alpha\dot\alpha}\overline{\Sigma} - \frac{1}{3}\ \overline{\mathrm{D}}\Sigma\ \mathrm{D}\overline{\Sigma}$$

$$\mathcal{J}^{min}_{\alpha\beta\dot\alpha\dot\beta} = \frac{1}{10}\ \partial^{(2)}\Sigma\ \overline{\Sigma} + \frac{1}{10}\ \Sigma\ \partial^{(2)}\overline{\Sigma} - \frac{2}{5}\ \partial\Sigma\ \partial\overline{\Sigma} + \frac{i}{5}\ \overline{\mathrm{D}}\Sigma\ \partial\mathrm{D}\overline{\Sigma} - \frac{i}{5}\ \partial\overline{\mathrm{D}}\Sigma\ \mathrm{D}\overline{\Sigma}$$

$$\mathcal{J}^{min}_{\alpha\beta\gamma\dot\alpha\dot\beta\dot\gamma} = \frac{i}{35}\ \partial^{(3)}\Sigma\ \overline{\Sigma} - \frac{i}{35}\ \Sigma\ \partial^{(3)}\overline{\Sigma} + \frac{9i}{35}\ \partial\Sigma\ \partial^{(2)}\overline{\Sigma} - \frac{9i}{35}\ \partial^{(2)}\Sigma\ \partial\overline{\Sigma}$$
$$+\frac{3}{35}\ \overline{\mathrm{D}}\Sigma\ \partial^{(2)}\mathrm{D}\overline{\Sigma} - \frac{9}{35}\ \partial\overline{\mathrm{D}}\Sigma\ \partial\mathrm{D}\overline{\Sigma} + \frac{3}{35}\ \partial^{(2)}\overline{\mathrm{D}}\Sigma\ \mathrm{D}\overline{\Sigma}\ .$$

Conservation Equation

- invariance of the action + $\overbrace{\mathrm{D}\Sigma = 0}^{\text{e.o.m}}$:

$$\overline{\mathrm{D}}^{\dot\alpha_{k+1}} \mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} = \frac{1}{(k+1)!}\overline{\mathrm{D}}^2 \mathrm{D}_{(\alpha_{k+1}} \mathcal{T}_{\alpha(k))\dot\alpha(k)}$$

$$\overline{\mathrm{D}}^2 \mathcal{J} = 0$$

- minimal supercurrent multiplet:

$$\overline{\mathrm{D}}^{\dot\alpha_{k+1}} \mathcal{J}^{min}_{\alpha(k+1)\dot\alpha(k+1)} = 0$$

- reality of $\mathcal{J}^{min}_{\alpha(k+1)\dot\alpha(k+1)}$ and the conservation equation fix a_p, b_p

$$\mathcal{J}^{min\ \Sigma}_{\alpha(s)\dot\alpha(s)} = \sum_{p=0}^{s} a_p\ \partial^{(p)}\Sigma\ \partial^{(s-p)}\overline{\Sigma}\ +\ \sum_{p=0}^{s-1} b_p\ \partial^{(p)}\overline{\mathrm{D}}\Sigma\ \partial^{(s-p-1)}\mathrm{D}\overline{\Sigma}$$

$$\mathcal{J}^{min\ \Sigma}_{\alpha(s)\dot\alpha(s)} = -\frac{(-i)^s}{\binom{2s+1}{s+1}} \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \left\{ \partial^{(p)}\Sigma\ \partial^{(s-p)}\overline{\Sigma} + i\left(\frac{s-p}{p+1}\right)\ \partial^{(p)}\overline{\mathrm{D}\Sigma}\ \partial^{(s-p-1)}\mathrm{D}\overline{\Sigma} \right\}$$

$$\mathcal{J}^{min\ \Phi}_{\alpha(s)\dot\alpha(s)} = -\frac{(-i)^s}{\binom{2s+1}{s+1}} \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \left\{ \partial^{(p)}\Phi\ \partial^{(s-p)}\overline{\Phi} + i\left(\frac{s-p}{p+1}\right)\ \partial^{(p)}\mathrm{D}\Phi\ \partial^{(s-p-1)}\overline{\mathrm{D}\Phi} \right\}$$

Chiral-complex linear duality

The parent action (σ - an unconstraint superfield, $\bar{D}\Phi = 0$, $\bar{D}^2\Sigma = 0$):

$$S_{pa} = -\int d^8z\bar{\sigma}\sigma + \int d^8z\ \Phi\sigma + \int d^8z\bar{\Phi}\bar{\sigma} + g\int d^8z\sum_{s=0}^{\infty} H^{\alpha(s)\dot{\alpha}(s)}\mathcal{J}^{min\ \Sigma}_{\alpha(s)\dot{\alpha}(s)}(\sigma,\bar{\sigma})$$

integrate out Φ integrate out σ

$$S = -\int d^8z\ \bar{\Sigma}\Sigma + g\int d^8z\sum_{s=0}^{\infty} H^{\alpha(s)\dot{\alpha}(s)}\mathcal{J}^{min\ \Sigma}_{\alpha(s)\dot{\alpha}(s)}\Big|_{\sigma=\Sigma}$$

$$S = \int d^8z\ \bar{\Phi}\ \Phi + g\int d^8z\sum_{s=0}^{\infty}(-1)^{s+1}H^{\alpha(s)\dot{\alpha}(s)}\mathcal{J}^{min\ \Phi}_{\alpha(s)\dot{\alpha}(s)}$$

$$g_s^{\Phi} = (-1)^{s+1}g_s^{\Sigma}$$

Chiral-complex linear duality

$$g_s^{\Phi} = (-1)^{s+1} g_s^{\Sigma}$$

$$\Big\downarrow \; j = s + 1$$

$$g_s^{\Phi} = (-1)^{j} g_s^{\Sigma}$$

- j is odd - attractive and repulsive force
- j is even - only attractive force
- the vector multiplet - $j = 1$, $g_s^{\Phi} = -g_s^{\Sigma}$
- supergravity - $j = 2$, $g_s^{\Phi} = g_s^{\Sigma}$

Summary

- Noether's method + 'HS' superdiffeomorphism - cubic interactions
- cubic interactions - the half-integer superspin supermultiplets
- for free theories - the minimal HS supercurrent multiplet
- the duality holds with an interesting twist: $g_s^{\Phi} = (-1)^{s+1} g_s^{\Sigma}$

Thank you for your attention!

$$
\begin{aligned}
X_{\alpha(k+1)\dot\alpha(k+1)} =& -\tfrac{i^{k+1}}{2}\left[\ \partial^{(k+1)}\Sigma\ \overline{\Sigma}\ + (-1)^{k+1}\ \Sigma\ \partial^{(k+1)}\overline{\Sigma}\ \right] \\
& -\tfrac{i^k}{2}\sum_{n=0}^{k}(-1)^n\Big\{\partial^{(k-n)}[\mathrm{D},\overline{\mathrm{D}}]\Sigma\ \partial^{(n)}\overline{\Sigma} + \partial^{(k-n)}\Sigma\ \partial^{(n)}[\mathrm{D},\overline{\mathrm{D}}]\overline{\Sigma}\Big\} \\
& -i^k\sum_{n=0}^{k}(-1)^n\left\{\partial^{(k-n)}\mathrm{D}\Sigma\ \partial^{(n)}\overline{\mathrm{D}}\overline{\Sigma}\ -\ \partial^{(k-n)}\overline{\mathrm{D}}\Sigma\ \partial^{(n)}\mathrm{D}\overline{\Sigma}\right\}\quad , \\
Z_{\alpha(k)\dot\alpha(k)} =& -i^k\sum_{n=0}^{k}(-1)^n\ \partial^{(k-n)}\Sigma\ \partial^{(n)}\overline{\Sigma} \\
Y_{\alpha(k)\dot\alpha(k)} =& -i^{(k-1)}\ \overline{\mathrm{D}}\left\{\sum_{n=0}^{k-1}(-1)^n\ \partial^{(k-1-n)}\mathrm{D}\Sigma\ \partial^{(n)}\overline{\Sigma}\right\}
\end{aligned}
$$