Coupled scalar perturbations of Galileon cosmologies in the mechanical approach in the late Universe

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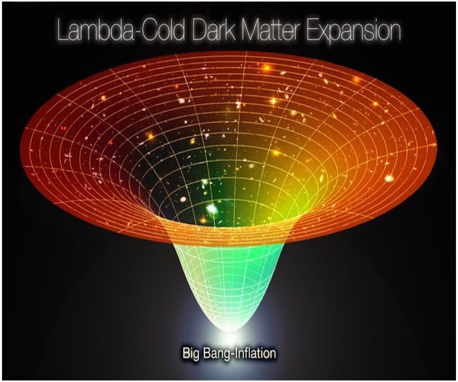
ΛCDM and cosmological constant

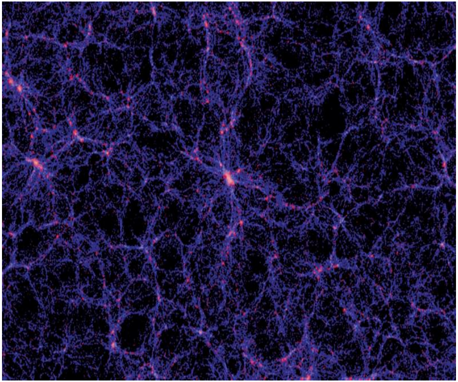


The ΛCDM model is consistent with observational data, but the energy scale of dark energy is too low.

Therefore this cosmological constant is not compatible with the cosmological constant originated from the vacuum energy in quantum field theory.

New field or modify gravity?

- One can assume that the dark energy is due to a new field.
- The other possibility is to modify the law of gravity from general relativity at large distances.



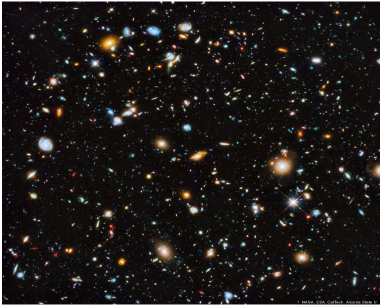
Galileon

Mostly inspired by DGP models, people derived the five Lagrangians that lead to field equations invariant under the Galilean symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in the Minkowski spacetime:

$$L_1 = M^3\phi,\ \ L_2 = (\nabla\phi)^2,\ \ L_3 = (\Box\phi)(\nabla\phi)^2/M^3,$$
$$L_4 = (\nabla\phi)^2[2(\nabla\phi)^2 - 2\phi_{;\mu\nu}\phi^{\mu\nu} - R(\nabla\phi)^2/2]/M^6,$$
$$L_5 = (\nabla\phi)^2[(\nabla\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\rho}\phi_{;\rho}{}^{\mu}$$
$$-6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}]/M^9,$$

The scalar field that respects the Galileon symmetry is the Galileon.

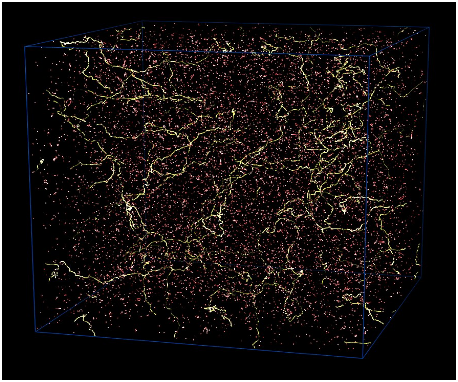
Mechanical approach



The mechanical approach works well for the ΛCDM model, where the peculiar velocities of the inhomogeneities could be considered as negligibly small, when we compare it with the speed of light. Additionally, we consider scales deep inside the cell of uniformity. Then we can drop the peculiar velocities at the first order of approximation.

The result for coupled Galileon field

At the background level, such Galilean field behaves as a 3-component perfect fluid: a network of cosmic strings with the EoS parameter $w = -\frac{1}{3}$, cosmological constant and some matter component.



Action

$$S_I = \alpha \int_M \sqrt{|g|}\, \Box\phi\, \partial_\mu\phi\, \partial^\mu\phi\, d^4x,$$

α is a small parameter, which measure the deviation from the model of minimally coupled scalar field and it has units L^3 (L is a length). First we will compute the tensor of energy momentum for this Lagrangian by the following formula:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$$

Metric

$$g^{\mu\nu} = \begin{pmatrix} \frac{1-2\Phi}{a^2} & 0 & 0 & 0 \\ 0 & -\gamma^{11}\frac{1+2\Psi}{a^2} & -\gamma^{12}\frac{1+2\Psi}{a^2} & -\gamma^{13}\frac{1+2\Psi}{a^2} \\ 0 & -\gamma^{21}\frac{1+2\Psi}{a^2} & -\gamma^{22}\frac{1+2\Psi}{a^2} & -\gamma^{23}\frac{1+2\Psi}{a^2} \\ 0 & -\gamma^{31}\frac{1+2\Psi}{a^2} & -\gamma^{32}\frac{1+2\Psi}{a^2} & -\gamma^{33}\frac{1+2\Psi}{a^2}. \end{pmatrix}$$

We use the case $K = 0$ and so

$$\gamma^{ij} = \gamma_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the trace $\gamma = \gamma^{ij}\gamma_{ij} = 3$.

D'Alembertian

We compute $\Box\phi$ with the perturbed quantities. We use the notation $\phi = \phi_c + \varphi$.

$$\Box\phi = \frac{\phi_c''}{a^2} - \frac{2}{a^2}\phi_c''\Phi + \frac{\varphi''}{a^2} - \frac{\Delta_{ij}\varphi}{a^2} -$$
$$\frac{\gamma}{a^2}\phi_c'(\frac{a'}{a} - \Psi' - 2\Phi\frac{a'}{a}) - \frac{a'}{a}\frac{\gamma}{a^2}\varphi' - -\frac{a'\phi_c'}{a^3} - \frac{\varphi' a'}{a^3} +$$
$$+\frac{4\Phi\phi_c' a'}{a^3} - \frac{\phi_c'}{a^3}(\Phi' a + 2a'\Phi) - \Box_{ij}\varphi\frac{1}{a^2}$$

Complete action

Now we wrote the tensor of energy-momentum for the whole action, when we include also the minimally coupled scalar field:

$$S = \int_M \sqrt{|g|} \, (\frac{1}{2} \, \partial_\rho \phi \, \partial^\rho \phi - V(\phi) + \alpha \, \Box \phi \, \partial_\mu \phi \, \partial^\mu \phi) \, d^4 x,$$

α is a small parameter, which measure the deviation from the model of minimally coupled scalar field and it has units L^3 (L is a length).

Einstein equations

So, the first Einstein equation is the following:

$$\Delta\Phi - 3H(\Phi' + H\Phi) + 3K\Phi = \frac{\kappa}{2}a^2(\delta\epsilon_{dust} + \delta\epsilon_{rad}) +$$
$$+\frac{\kappa}{2}[-(\phi_c')^2\Phi + \phi_c'\varphi' + a^2\frac{dV}{d\phi}(\phi_c)\varphi +$$
$$2\alpha\,\gamma\frac{(\phi_c')^2}{a^2}(\phi_c'\Psi' + 4\Phi\frac{a'}{a}\phi_c' - \frac{3\varphi' a'}{a}) -$$
$$-\alpha\,\frac{2(\phi_c')^2}{a^2}\Box_{ij}\varphi]$$

EoM

We use now the equation of motion:

$$-2\alpha(\Box\phi)^2 + 2\alpha\nabla^\mu\nabla^\nu\phi\ \nabla_\mu\nabla_\nu\phi + 2\alpha\nabla^\mu\phi\nabla^\nu\phi\ R_{\mu\nu} -$$

$$-\Box\phi - \frac{dV}{d\phi} = 0$$

Peculiar velocity of the scalar field

When we consider the mechanical approach, we can drop the terms containing the peculiar velocities of the inhomogeneities and radiation as these are negligible when compared with their respective energy density and pressure fluctuations. If we deal with a scalar field, such an approach is not evident since the quantity treated as the peculiar velocity of the scalar field is proportional to the scalar field perturbation φ.

Work with Einstein equations

$$\frac{\delta\epsilon_{RAD}}{3} = \frac{-1}{3a^6}[(+36\alpha H\phi_c'\phi_c''a^2 + 18\alpha H^2(\phi_c')^2a^2 + 18\alpha H'(\phi_c')^2a^2 +$$
$$+12\alpha\phi_c''a\phi_c'a' + 18\alpha(\phi_c')^2a''a -$$
$$-18\alpha(\phi_c')^2(a')^2)\varphi - 30\alpha\Phi(\phi_c')^3a'a - 6\alpha\Phi(\phi_c')^2\phi_c''a^2 +$$
$$(36\alpha(\phi_c')^2a'a)\varphi' + 3\Phi a^6\overline{\epsilon}_{DUST} + 4\Phi a^6\overline{\epsilon}_{RAD}]$$

As matter sources, we also include dust-like matter (baryonic and CDM) and radiation. The background (average) energy density of the dust-like matter takes the form $\overline{\epsilon}_{DUST} = \overline{\rho}c^2/a^3$, where $\overline{\rho} = const.$ is the average comoving rest mass density. For radiation we have the EoS $\overline{p}_{RAD} = \frac{1}{3}\overline{\epsilon}_{RAD}$ and $\overline{\epsilon}_{RAD} \sim 1/a^4$.

Pure scalar field

We get for the case of pure scalar field the following:

$$\Phi[-\frac{2}{3}a^2\kappa\bar{\epsilon}_{RAD}-\frac{1}{2}a^2\kappa\bar{\epsilon}_{DUST}]=\kappa\frac{a^2}{2}\delta p_{RAD}$$

$$-\Phi\overline{\rho}c^2/a^3=\frac{1}{3}\delta\epsilon_{RAD}$$

Central equation for pure scalar field

Next we make the substitution $\Phi = \Omega/a$ in the following equation for the pure scalar field:

$$\Delta\Phi - \frac{\kappa}{2}\frac{\delta\rho c^2}{a} = \Phi[3H^2 - 2K - \frac{\kappa}{2}(\phi_c')^2 + H' - -H\frac{\phi_c''}{\phi_c'} + a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] + \frac{d\Phi}{da}a[5H^2 + H' - H\frac{\phi_c''}{\phi_c'} + +a^2\frac{dV}{d\phi}(\phi_c)\frac{H}{\phi_c'}] + \frac{d^2\Phi}{da^2}H^2a^2$$

Application of mechanical approach

The dust like matter component is considered in the form of discrete distributed inhomogeneities. Then we are looking for solutions of previous equation, which have a Newtonian limit near gravitating masses. Such an asymptotic behaviour will take place if we impose $\Omega = \Omega(\vec{r})$.

$$\Omega = \Omega(\vec{r}) \Rightarrow \Phi \sim \frac{1}{a} \text{ and } (\phi_c')^2 = const.$$

Let's denote $\phi_c' = \beta = const.$, then we get

$$\phi_c = \beta\eta + \gamma, \;\; \gamma = const.$$

$$V = \frac{\beta^2}{a^2} + V_\infty$$

Galileon cosmologies and mechanical approach

Now, we suppose that $\Omega = \Omega(r)$, which means that $\phi_c' = const.$:

$$\phi_c(\eta) = \beta\eta + \omega,$$

where ω and β are constants.
Then we get from the equation of motion that

$$6\alpha\beta^2 a'' + 2a^2 a'\beta + \frac{dV}{d\phi} a^5 = 0$$

Potential

We want to obtain the dependence $f(\eta)$ in the relation

$$V(\eta) = \frac{\beta^2}{a^2} + V_\infty + \alpha f(a),$$

because we know the dependence $V(a) = \frac{\beta^2}{a^2} + V_\infty$ for the pure scalar field.

$f(a)$ behaves like matter:

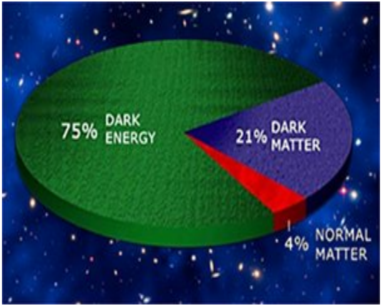
$$f(a) \sim \frac{1}{a^3}$$

K-essence models

$$S = \int \sqrt{|g|}\ P(X,\phi)\ d^4x, \tag{1}$$

$X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi.$

PKK can provide cosmic acceleration.



Articles

This presentation was prepared with the help of

- Maxim Eingorn, J.N., Alexander Zhuk, f(R) gravity: scalar perturbations in the late Universe, EPJ C
- Mariam Bouhmadi-Lopéz, J.N., Coupled scalar perturbations of Galileon cosmologies in the mechanical approach in the late Universe, in preparation
- Alexander Zhuk, Perfect fluids coupled to inhomogeneities in the late Universe
- Alvina Burgazli, Alexander Zhuk, João Morais, Mariam Bouhmadi Lopéz, K.Sravan Kumar, Coupled scalar fields in the late Universe: The mechanical approach and the late time cosmic acceleration
- Mariam Bouhmadi-Lopéz, K.Sravan Kumar, João Marto, João Morais, Alexander Zhuk, K-essence model from the mechanical approach point of view: coupled scalar field and the late time cosmic acceleration

Sources of pictures: Arizona State University, Backreaction: blogger, Discovery magazine blog, NASA Getty Images, The University of Chicago, Cosmology: Brian Koberlein, New Scientist