$\widehat{\mathfrak{u}}(1)$-breaking branes from minimal models

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(work in progress)

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2d CFTs with boundary

- conformally invariant QFTs on Riemann surfaces with boundaries (UHP)
  - $\rightarrow$ world-sheet description of D-branes in *(super)string theory*
  - $\rightarrow$ impurities, defects, point-contacts, ... in *condensed matter*
- boundary states: preserve 1/2 of bulk conformal symmetry

$$(L_n - \overline{L}_{-n})\|B\rangle\rangle = 0$$

asymptotic states in the full-plane CFT Hilbert space

- *solution:* Ishibashi states $|V_i\rangle\rangle$ in 1–1 correspondence with spinless bulk primaries

$$\|B\rangle\rangle = \sum_i g_i^B |V_i\rangle\rangle$$

- consistency on genus 0 and 1: Cardy-Lewellen sewing conditions
  - $\rightarrow$ constraints on g_i^B and other bulk-boundary and boundary couplings
  - $\rightarrow$ genus one (Cardy)

$$Z_{AB}(q) = \mathrm{Tr}_{\mathcal{H}^{(\mathrm{B})}} q^{L_0^{(\mathrm{B})} - \frac{c}{24}} = \langle\langle B\| \tilde{q}^{\frac{1}{2}(L_0^{(\mathrm{H})} + \overline{L}_0^{(\mathrm{H})} - \frac{c}{12})} \|A\rangle\rangle$$

where $\tilde{q}(\tau) = q(-1/\tau)$

Boundary states for free bosons

- D compactified free bosons, $\alpha' = 1$, $w \in \Lambda$, $k \in \Lambda^*$, $k_{\mathrm{L,R}} = k \pm w$

$$Z_{\mathrm{b}}(q,\bar{q}) = \frac{1}{|\eta(q)|^{2D}} \sum_{w\in\Lambda, k\in\Lambda^*} q^h \bar{q}^{\bar{h}}\,,$$

  where $h = k_{\mathrm{L}}^2/4$, $\bar{h} = k_{\mathrm{R}}^2/4$
- *not* rational $\rightarrow$ impose additional $\widehat{\mathfrak{u}}(1)$-symmetry

$$(j^\mu_n + \Omega^\mu{}_\nu \bar{j}^\nu_{-n})\|B\rangle\rangle = 0$$

  such that $\Omega^\mu{}_\rho \Omega^\nu{}_\sigma g^{\rho\sigma} = g^{\mu\nu}$. Gives $\mathrm{D}p$ branes with $U(1)$ fluxes.
- *closed string theory:* $\|B\rangle\rangle$ sources closed string fields $\rightarrow$ effective actions
  - energy density:

$$\begin{aligned} E_B(x,\tilde{x}) \quad &\propto \quad \int dk_0\, e^{ik_0x^0} \sum_{w\in\Lambda, k\in\Lambda^*} \langle k, w; k^0|\alpha_1^0\overline{\alpha}_1^0\|B\otimes N^0\rangle\rangle e^{ik\cdot X + iw\cdot\tilde{X}} \\ &\propto \quad \sum_{w\in\Lambda, k\in\Lambda^*} \langle k, w\|B\rangle\rangle e^{ik\cdot x+iw\cdot\tilde{x}} \end{aligned}$$

    e.g. $E_{\mathrm{D}0}(x,\tilde{x}) \propto g_{D0}\,\delta^D(x-x_0)$
- *open string theory:* dynamics of open string excitations described by boundary fields and BCCOs

- there are branes with $(j_n^\mu + \Omega^\mu{}_\nu \bar{j}^\nu_{-n})\|B\rangle\rangle \neq 0$: in general we need only

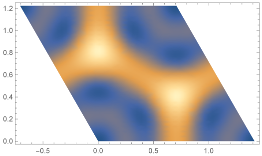
$$(L_n - \overline{L}_{-n})\|B\rangle\rangle = 0\,.$$

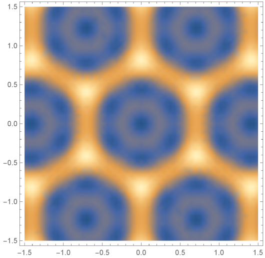
$\rightarrow$ e.g. $SU(2)$ marginal deformations of $c=1$ circle at $R=(m/n)R_{\mathrm{s.d.}}$

$$m\,\mathrm{D0}\text{ branes} \longrightarrow n\,\mathrm{D1}\text{ branes}$$

solution: organise primaries into $SU(2)$ multiplets (Gaberdiel et al.)

- exotic solutions of tachyon condensation in bosonic OSFT: branes wrapping A_2 root-lattice compactification (M. Kudrna)
  - $\rightarrow$ KK modes on D2 brane
  - $\rightarrow$ stretched strings in a system of two D0 branes (or $\mathrm{D0}$-$\overline{\mathrm{D0}}$ in IIB)





Rationalizing free boson CFTs

- *strategy:* extend $\mathcal{V}ir_{\rm b}$ by higher-spin bosonic currents, such that $\mathcal{A}_{\rm ext} = \{W^k(z)\,|\,k\}$ is rational and $\mathcal{A}_{\rm b} = \{j^\mu(z)\,|\,\mu = 1,\ldots,D\} \notin \mathcal{A}_{\rm ext}$
- typically $\mathcal{A}_{\rm ext} = \mathcal{A}_1 \oplus \mathcal{A}_2 \oplus \ldots \oplus \mathcal{A}_N$, N finite, $\mathcal{A}_k$ unitary rational chiral algebras, s.t. $\mathcal{A}_k \supset \mathcal{V}ir_k = \{T_k(z)\}$ with $c_{\rm b} = D = \sum_k c_k$
- it follows that

$$\mathcal{H}_{\rm b} = \bigoplus_{i,\bar\imath} M_{i\bar\imath}\,(\mathcal{H}^1_{i_1} \otimes \ldots \otimes \mathcal{H}^N_{i_N}) \otimes (\overline{\mathcal{H}}^1_{\bar\imath_1} \otimes \ldots \otimes \overline{\mathcal{H}}^N_{\bar\imath_N})$$

multiindices $i = (i_1,\ldots,i_N)$ and $\bar\imath = (\bar\imath_1,\ldots,\bar\imath_N)$ run over the irreducible modules of the unitary algebras $\mathcal{A}_k$, $M_{00} = 1$, M modular invariant over $\mathcal{A}_{\rm ext} \oplus \overline{\mathcal{A}}_{\rm ext}$. Equivalently

$$Z_{\rm b}(q,\bar q) = \sum_{i,\bar\imath} \chi^1_{i_1}(q)\chi^2_{i_2}(q)\ldots\chi^N_{i_N}(q)\,M_{i\bar\imath}\,\bar\chi^1_{\bar\imath_1}(\bar q)\bar\chi^2_{\bar\imath_2}(\bar q)\ldots\bar\chi^N_{\bar\imath_N}(\bar q)\,.$$

- $\mathcal{A}_{\rm ext} \oplus \overline{\mathcal{A}}_{\rm ext}$ primaries

$$W^k_{s_k}(z)\,\phi(w,\bar w) \quad = \quad \frac{w^k_{s_k}\,\phi(w,\bar w)}{(z-w)^{s_k}} + \mathcal{O}[(z-w)^{-s_k+1}]\,,$$

- works only at isolated points in bulk moduli space

Boundary states in RCFTs

- (unitary) rational chiral algebra $\mathcal{A} = \{W_k(z) \,|\, k\}$, modular $\mathcal{S}$-matrix S_{ij} & modular-invariant mass matrix $M_{i\bar{\imath}}$ give RCFT $\mathfrak{R} = (\mathcal{A}, M_{i\bar{\imath}})$ with

$$Z_{\mathfrak{R}}(q, \bar{q}) = \sum_{i, \bar{\imath} \in \mathcal{I}(\mathcal{A})} \chi_i^{\mathcal{A}}(q)\, M_{i\bar{\imath}}\, \overline{\chi}_{\bar{\imath}}^{\mathcal{A}}(\bar{q})\,.$$

- gluing conditions

$$(W_n^i - (-1)^{|W^i|}\Omega \overline{W}_{-n}) \| B \rangle\rangle = 0$$

such that $(L_n - \overline{L}_{-n})\| B \rangle\rangle = 0$, let $\mathcal{E}_\omega(\mathfrak{R}) = \{ i \in \mathcal{I}(\mathcal{A}) \,|\, i = \omega(i^+) \}$

- can classify all fundamental independent boundary states for fixed ω (BPPZ, 1998): have $Z_{AB}(q) = \sum_i {n_{iA}}^B \chi_i^{\mathcal{A}}$ with $n_i \in \mathrm{Mat}_{|\mathcal{E}_\omega|}(\mathbb{Z}_{>0})$

$$n_i n_j = \sum_{k \in \mathcal{I}(\mathcal{A})} {N_{ij}}^k n_k$$

$\rightarrow$ need to find NIM-reps of fusion algebra, then

$${n_{iA}}^B = \sum_{j \in \mathcal{E}_\omega} \frac{S_{ij}}{S_{0j}} (\psi_A^j)^* \psi_B^j$$

with $\| B \rangle\rangle = \sum_{j \in \mathcal{E}_\omega} \frac{\psi_B^j}{\sqrt{S_{0j}}}$, i.e. ψ_A^j are simultaneous ON eigenvectors of ${n_{iA}}^B$

- if $\mathcal{E}_\omega = \mathcal{I}$, then $\psi_A^j = S_{Aj}$ with ${n_{iA}}^B = {N_{iA}}^B$ (Cardy branes)

Toy example I: $c = 1$ at $R = \sqrt{2\alpha'}$

- take $\mathcal{A}_{\mathrm{ext}} = \{T_1(z), T_2(z)\} \cong \mathcal{V}ir^{c=\frac{1}{2}} \oplus \mathcal{V}ir^{c=\frac{1}{2}}$, where

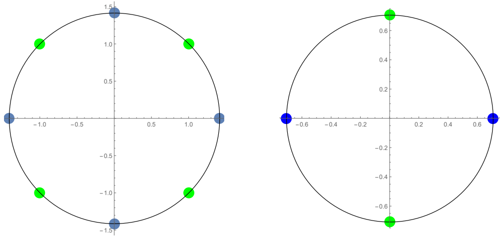
$$T_{1,2}(z) \quad = \quad -\frac{1}{2} :j(z)j(z): \pm \frac{1}{4}\left(e^{2i\sqrt{2}X_{\mathrm{L}}(z)} + e^{-2i\sqrt{2}X_{\mathrm{L}}(z)}\right)$$

- torus partition function

$$Z_{\mathrm{b}}(q,\bar{q}) \quad = \quad |\chi_0\chi_0 + \chi_{\frac{1}{2}}\chi_{\frac{1}{2}}|^2 + |\chi_0\chi_{\frac{1}{2}} + \chi_{\frac{1}{2}}\chi_0|^2 + 2|\chi_{\frac{1}{16}}\chi_{\frac{1}{16}}|^2$$

- gluing conditions

$$[(L_{1,2})_n - (\overline{L}_{1,2})_{-n}]|i\rangle\!\rangle_{\mathrm{id}} = 0\,, \quad [(L_{1,2})_n - (\overline{L}_{2,1})_{-n}]|i\rangle\!\rangle_{\pi} = 0\,,$$



(a) $R = \sqrt{2}$.

(b) $\tilde{R} = 1/\sqrt{2}$.

Toy example II: $\mathbb{S}^1/\mathbb{Z}_2$ orbifold at $R = \sqrt{2\alpha'}$

- project onto fields even under $X \to -X$: $T_{1,2}(z)$ are even $\to \mathcal{A}_{\rm ext}$ survives; need to modify mass matrix to reflect emergence of twisted sectors
- untwisted sector

$$Z_{\rm u}(q,\bar{q}) \;=\; |\chi_0\chi_0|^2 + |\chi_{\frac{1}{2}}\chi_{\frac{1}{2}}|^2 + |\chi_0\chi_{\frac{1}{2}}|^2 + |\chi_{\frac{1}{2}}\chi_0|^2 + |\chi_{\frac{1}{16}}\chi_{\frac{1}{16}}|^2 .$$

with $Z_{\rm orb} = Z_{\rm u} + \mathcal{S}(Z_{\rm u}) + \mathcal{TS}(Z_{\rm u}) - Z_{\rm b}$ gives

$$Z_{\rm orb}(q,\bar{q}) \;=\; \left(|\chi_0|^2 + |\chi_{\frac{1}{2}}|^2 + |\chi_{\frac{1}{16}}|^2\right)^2 .$$

$\to \mathrm{Ising}^2$

- boundary states
  - id automorphism type: 4 fractional D0s, 4 fractional D1s, 1 bulk D0
  - π automorphism type: 2 bulk D0s, 1 bulk D0

correspond to 9 Cardy branes and 3 permutation branes of Ising^2

A_2 root-lattice at $R = \sqrt{2\alpha'}$: chiral algebras

- smallest possible extension $\mathcal{A}_{\mathrm{ext}} = \mathcal{W}_3^{c=\frac{4}{5}} \oplus \mathcal{W}_{2,2,3}^{c=\frac{6}{5}}$ (check: $2 = \frac{4}{5} + \frac{6}{5}$)
- have $\mathcal{W}_3^{c=\frac{4}{5}} = \{T_{\frac{4}{5}}(z), W_{\frac{4}{5}}(z)\}$ with (here a_i simple roots, b^i fund. weights)

$$
\begin{aligned}
T_{\frac{4}{5}}(z) &= -\frac{2}{5}[:j^1 j^1: + :j^2 j^2: - \sum_{i=1}^{3} \cos(2a_i \cdot X_{\mathrm{L}})]\,, \\
W_{\frac{4}{5}}(z) &= \frac{2i}{\sqrt{65}} :[2(\frac{1}{3} j^1 j^1 j^1 - j^1 j^2 j^2) + \sqrt{6} \sum_{i=1}^{3} (b^i \cdot j) \cos(2a_i \cdot X_{\mathrm{L}})] :
\end{aligned}
$$

- also $\mathcal{W}_{2,2,3}^{c=\frac{6}{5}} = \{T_{\frac{6}{5}}(z), W_{\frac{6}{5}}(z), U^{\pm}_{\frac{6}{5}}(z)\} = \mathcal{W}_3^{c=\frac{6}{5}} \cup \{U^{\pm}_{\frac{6}{5}}(z)\}$ with $(\omega = e^{+\frac{2i\pi}{3}})$

$$
\begin{aligned}
T_{\frac{6}{5}}(z) &= -\frac{3}{5}[:j^1 j^1: + :j^2 j^2: + \frac{2}{3} \sum_{i=1}^{3} \cos(2a_i \cdot X_{\mathrm{L}})]\,, \\
W_{\frac{6}{5}}(z) &= \frac{2}{\sqrt{35}} :[\frac{i}{\sqrt{2}} (j^2 \partial j^1 - j^1 \partial j^2) + \sqrt{6} \sum_{i=1}^{3} (b^i \cdot j) \sin(2a_i \cdot X_{\mathrm{L}})] : \,. \\
U^{\pm}_{\frac{6}{5}}(z) &= -\frac{3}{4} :(j^1 \mp i j^2)^2: + \sum_{i=1}^{3} \omega^{\pm(i-1)} \cos(2a_i \cdot X_{\mathrm{L}})\,,
\end{aligned}
$$

A_2 root-lattice at $R = \sqrt{2\alpha'}$: chiral algebras

- chiral algebra $\mathcal{W}^{c=\frac{6}{5}}_{2,2,3}$ indeed closes:

$$
\begin{aligned}
U^{\pm}_{\frac{6}{5}}(z)U^{\pm}_{\frac{6}{5}}(w) &= \frac{2U^{\mp}_{\frac{6}{5}}(w)}{(z-w)^2} + \frac{\partial U^{\mp}_{\frac{6}{5}}(w)}{z-w} + \mathcal{O}[(z-w)^0]\,, \\
U^{+}_{\frac{6}{5}}(z)U^{-}_{\frac{6}{5}}(w) &= \frac{7}{8}\left\{\frac{3}{(z-w)^4} + 5\left[\frac{2T_{\frac{6}{5}}(w)}{(z-w)^2} + \frac{\partial T_{\frac{6}{5}}(w)}{z-w}\right] + \frac{45\gamma W_{\frac{6}{5}}(w)}{z-w}\right\} \\
&\qquad + \mathcal{O}[(z-w)^0]\,,
\end{aligned}
$$

- $\mathcal{W}^{c=\frac{6}{5}}_{2,2,3}$ has equivalent description in terms parafermion $\mathcal{W}^{c=\frac{6}{5}}_{2,2,3} \cong \frac{\widehat{\mathfrak{su}}(3)_2}{\widehat{\mathfrak{u}}(1)^2_2}$ with currents

$$
\psi_{\frac{1}{2},i}(z) \propto \sin(a_i \cdot X_{\mathrm{L}})
$$

for $i = 1,2,3$, while $\mathcal{W}^{c=\frac{4}{5}}_{3} \cong \frac{\widehat{\mathfrak{su}}(2)_3}{\widehat{\mathfrak{u}}(1)_3}$ with currents

$$
\psi_{\frac{2}{3}}(z) \propto \sum_{i=1} e^{2ib^i \cdot X_{\mathrm{L}}}\,, \qquad \psi^{\dagger}_{\frac{2}{3}}(z) \propto \sum_{i=1}^{3} e^{-2ib^i \cdot X_{\mathrm{L}}}
$$

A_2 root-lattice at $R = \sqrt{2\alpha'}$: partition function

- denote by $\widehat{\mathfrak{u}}(1)^2_2$ the rational gaussian model $\{j^\mu(z), e^{\pm 2ia^i \cdot X_{\rm L}}\}$ on A_2 lattice

$$Z_{\rm b}(q,\bar{q}) = \sum_{k,\bar{k}\in\Lambda^*/2\Lambda} \chi_k^{\widehat{\mathfrak{u}}(1)^2_2}\, M^{\mathfrak{A}_2}_{k\bar{k}}\, \overline{\chi}_{\bar{k}}^{\widehat{\mathfrak{u}}(1)^2_2}$$

  with $\chi_k^{\widehat{\mathfrak{u}}(1)^2_2}(q) = \frac{1}{\eta(q)^2}\sum_{\rho\in 2\Lambda} q^{(k+\rho)^2/4}$ for $k \in \Lambda^*/2\Lambda$

- it can be shown that

$$\widehat{\mathfrak{u}}(1)^2_2 = \frac{\widehat{\mathfrak{su}}(3)_1 \oplus \widehat{\mathfrak{su}}(3)_1}{\widehat{\mathfrak{u}}(1)^2_2} = \frac{\widehat{\mathfrak{su}}(3)_2}{\widehat{\mathfrak{u}}(1)^2_2} \oplus \frac{\widehat{\mathfrak{su}}(3)_1 \oplus \widehat{\mathfrak{su}}(3)_1}{\widehat{\mathfrak{su}}(3)_2}$$

  with branching rules

$$\begin{aligned}
\chi_0^{\widehat{\mathfrak{u}}(1)^2_2} &= \chi_{0,0}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{0,0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} + \chi_{\frac{2}{5},0}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{\frac{3}{5},0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}}\,,\\
\chi_{\pm b^i}^{\widehat{\mathfrak{u}}(1)^2_2} &= \chi_{\frac{1}{15},\pm\alpha}^{\mathcal{W}_3^{c=\frac{4}{5}}}\chi_{\frac{1}{10},\lambda_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} + \chi_{\frac{2}{3},\mp\beta}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{\frac{1}{2},\mu_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}}\,,\\
\chi_{a_i}^{\widehat{\mathfrak{u}}(1)^2_2} &= \chi_{0,0}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{\frac{1}{2},\mu_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} + \chi_{\frac{2}{5},0}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{\frac{1}{10},\lambda_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}}\,,\\
\chi_{\pm 2b^1}^{\widehat{\mathfrak{u}}(1)^2_2} &= \chi_{\frac{1}{15},\pm\alpha}^{\mathcal{W}_3^{c=\frac{4}{5}}}\chi_{\frac{3}{5},0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} + \chi_{\frac{2}{3},\mp\beta}^{\mathcal{W}_3^{c=\frac{4}{5}}}\,\chi_{0,0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}}\,.
\end{aligned}$$

- can form larger extensions by embeddings, e.g. $\mathcal{W}_{2,2,3}^{c=\frac{6}{5}} \supset \mathcal{W}_{3}^{c=\frac{6}{5}}$

$$\begin{aligned}
\chi_{0,0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} &= \chi_{0,0}^{\mathcal{W}_{3}^{c=\frac{6}{5}}} + \chi_{2,+6\gamma}^{\mathcal{W}_{3}^{c=\frac{6}{5}}} + \chi_{2,-6\gamma}^{\mathcal{W}_{3}^{c=\frac{6}{5}}}\,, \\
\chi_{\frac{3}{5},0}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} &= \chi_{\frac{3}{5},+\gamma}^{\mathcal{W}_{3}^{c=\frac{6}{5}}} + \chi_{\frac{3}{5},-\gamma}^{\mathcal{W}_{3}^{c=\frac{6}{5}}} + \chi_{\frac{8}{5},0}^{\mathcal{W}_{3}^{c=\frac{6}{5}}}\,, \\
\chi_{\frac{1}{10},\lambda_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} &= \chi_{\frac{1}{10},0}^{\mathcal{W}_{3}^{c=\frac{6}{5}}}\,, \\
\chi_{\frac{1}{2},\mu_i}^{\mathcal{W}_{2,2,3}^{c=\frac{6}{5}}} &= \chi_{\frac{1}{2},0}^{\mathcal{W}_{3}^{c=\frac{6}{5}}}\,,
\end{aligned}$$

similarly we have

- $\mathcal{W}_{2,2,3}^{c=\frac{6}{5}} \supset \mathcal{V}ir^{c=\frac{1}{2}} \oplus \mathcal{V}ir^{c=\frac{7}{10}}$
- $\mathcal{W}_{3}^{c=\frac{4}{5}} \supset \mathcal{V}ir^{c=\frac{4}{5}}$

A_2 root-lattice at $R=\sqrt{2\alpha'}$: boundary states

- admissible gluing automorphisms for $\mathcal{W}_{\frac{4}{5}}^{c=\frac{4}{5}} \oplus \mathcal{W}_{2,2,3}^{c=\frac{6}{5}}$

$$\begin{aligned}
[(W_{\frac{4}{5}})_n + \Omega_{\frac{4}{5}}(\overline{W}_{\frac{4}{5}})_{-n}]\|B\rangle\rangle &= 0\\
[(W_{\frac{6}{5}})_n + \Omega_{\frac{6}{5}}(\overline{W}_{\frac{6}{5}})_{-n}]\|B\rangle\rangle &= 0\\
[(U^{\pm}_{\frac{6}{5}})_n - \Omega^{\pm}_{\frac{6}{5}}(\overline{U}^{\pm}_{\frac{6}{5}})_{-n}]\|B\rangle\rangle &= 0
\end{aligned}$$

with $\Omega_{\frac{4}{5}} \in \{+1,-1\}$, $\Omega_{\frac{6}{5}} \in \{+1,-1\}$, $\Omega^{\pm}_{\frac{6}{5}} = e^{\pm\frac{2ik\pi}{3}}$ for $k=0,1,2$
$\rightarrow$ these correspond to the 12 elements of D_6 acting on the compactification lattice

- brane spectrum

$$\begin{array}{cccccc}
\mathrm{D0} & \mathrm{D2}(\pm 3) & \mathrm{D2}(\pm 1) & \mathrm{D2}(0) & \mathrm{D1_S}(0,\pm\frac{2i\pi}{3}) & \mathrm{D1_L}(0,\pm\frac{2i\pi}{3})\\
\varphi\mathrm{D0} & \varphi\mathrm{D2}(\pm 3) & \varphi\mathrm{D2}(\pm 1) & \varphi\mathrm{D2}(0) & \varphi\mathrm{D1_S}(0,\pm\frac{2i\pi}{3}) & \varphi\mathrm{D1_L}(0,\pm\frac{2i\pi}{3})
\end{array}$$

where φ-branes can be obtained by fusing $\frac{2}{5}$-Potts' topological defect onto their partner Dp-branes

$$\text{tension of } \varphi\text{-brane} \quad = \quad \frac{1+\sqrt{5}}{2} \times \text{tension of } \mathrm{D}p \text{ brane}$$

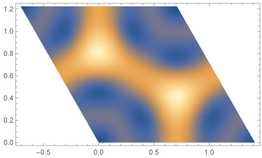
- can compute boundary states in terms of $\mathcal{W}_3^{c=\frac{4}{5}} \oplus \mathcal{W}_{2,2,3}^{c=\frac{6}{5}}$ Ishibashi states

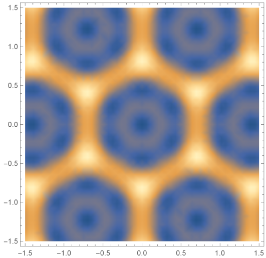
φD0 equals honeycomb!

- to draw profiles, need to express $\mathcal{W}_3^{c=\frac{4}{5}} \oplus \mathcal{W}_{2,2,3}^{c=\frac{6}{5}}$ Ishibashi states in terms of $\mathcal{V}ir_{\mathrm{b}}$ Ishibashi states and read of coefficients in front of $|k, w\rangle$
- can compare coefficients of φD0 with numerical solution of OSFT for tachyon condensation on D2 (M. Kudrna)

	S	E_0	E_{b^i}	E_{a_i}	E_{2b^i}	$E_{b^1+2b^2}$
OSFT (lev 14)	1.618030	1.61812	-0.61800	0.5008	0.125	-0.240
RCFT (decimal)	1.618034	1.61803	-0.61803	0.5000	0.127	-0.245
RCFT (exact)	$\frac{1+\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	$\frac{1}{2}$	$\frac{3-\sqrt{5}}{6}$	$\frac{3-2\sqrt{5}}{6}$

$\rightarrow$ honeycomb solution matches with φD0!





Summary, discussion and outlook

- for $c > 1$, free boson CFT admits boundary conditions which break the $\widehat{\mathfrak{u}}(1)$-current symmetry
- these can be found as solitonic solutions (tachyon lumps) in OSFT on various open string backgrounds
- corresponding boundary states can be computed analytically at certain points in bulk moduli space by rationalizing $\mathcal{V}ir_{\mathrm{b}}$

- existence of honeycomb state known already to Yi & Kane, 1997 (bdy RG flows) and Affleck, Oshikawa, Saleur, 2000 (boundary embedding, fusion)
- extensions of $\mathcal{V}ir_{\mathrm{b}}$ by minimal model Virasoros known already to Dong, Li, Mason, Norton, 1996 (Niemeier lattices, Moonshine module)

- improve energy profiles: calculate higher (possibly all) harmonics
- find boundary structure constants: open string interpretation?
- higher dimensional compactifications: $n-1 = \mathrm{rank}(\mathfrak{g})$ bosons on root-lattice of $\widehat{\mathfrak{g}} = A, D, E$ give $\mathcal{A}_{\mathrm{ext}} = \mathcal{P}_2^{\mathfrak{g}} \oplus \mathcal{W}(\mathfrak{g})^1$, $\mathcal{W}(\mathfrak{g})^1 = \frac{\widehat{\mathfrak{g}}_1 \oplus \widehat{\mathfrak{g}}_1}{\widehat{\mathfrak{g}}_2}$ and $\mathcal{P}_2^{\widehat{\mathfrak{g}}} = \frac{\widehat{\mathfrak{g}}_2}{\widehat{\mathfrak{u}}(1)_2^{n-1}}$. For $\mathfrak{g} = \mathfrak{su}(n)$ obtain $\mathcal{P}_2^{\widehat{\mathfrak{su}}(n)} \supset \mathcal{V}ir^3 \oplus \mathcal{V}ir^4 \oplus \ldots \oplus \mathcal{V}ir^{n+1}$.
- generalize to 0A/0B or IIA/IIB superstring

Thank you!