# THE 42<sup>th</sup> WINTER SCHOOL GEOMETRY AND PHYSICS

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# LIST OF PARTICIPANTS

Spyridon Afentoulidis Almpanis Sandipan Bhattacherjee Temesgen Tsegaye Bihonegn Eugenia Boffo Leron Borsten Ugo V. Boscain Frederic Bourgeois Andreas Čap Alessandro Carotenuto Goce Chadzitaskos Darek Cidlinský Carlo Alberto Cremonini Florian De Leger Georgios Dimitroglou Rizell Martin Doležal Maciej Dunajski Zdeněk Dušek Keegan Flood Roman Golovko Jan Gregorovič Erlend Grong Zhangwen Guo Anita Gurung Pavel Hájek **Christoph Harrach** Stanislav Hronek Štěpán Hudeček Josef Janyška Branislav Jurčo Igor Khavkine Martin Kolář Lukáš Krump Andrey Krutov Svatopluk Krýsl

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## ANNOUNCED LECTURES

### A. INVITED LECTURES

Leron Borsten: From gluons to gravitons via homotopy algebras Ugo V. Boscain: Diffusion in almost Riemannian geometry Frederic Bourgeois: Homological invariants of Legendrian submanifolds Georgios Dimitroglou Rizell: DG-Algebraic Aspects of Contact Invariants Erlend Grong: Curvature in sub-Riemannian geometry Konrad Waldorf: Gerbes, stacks, and strings

## B. Other lectures

Spyridon Afentoulidis Almpanis: Noncubic Dirac operators for finite dimensional modules Sandipan Bhattacherjee: Quantum Cosmology with a distinct hint of condensed matter physics **Eugenia Boffo**: *BRST of the* N = 2 *superparticle and R*-*R fields* Andreas Čap: Tractors and mass of asymptotically hyperbolic manifolds Alessandro Carotenuto: Principal pair of quantum homogeneous spaces Goce Chadzitaskos: Some interesting properties of an asymmetric oscillator Carlo Alberto Cremonini: Cohomology of Lie Superalgebras Martin Doležal: Growth vectors via forms Maciej Dunajski: Conformal Mercator Equation **Zdeněk Dušek**: Geodesic lemma for Finsler ( $\alpha$ ,  $\beta$ ) metrics and some special Finsler q.o. manifolds **Keegan Flood**: The jet functor in non-commutative geometry Roman Golovko: On infinite number of Hamiltonian non-isotopic exact Lagrangian fillings in high dimensions Jan Gregorovič: Construction of counterexamples to the 2-jet determination Chern-Moser Theorem in higher codimension Zhangwen Guo: Weyl structures for path geometries Christoph Harrach: On boundary values of Poisson transforms for differential forms **Stanislav Hronek**: The story of O(D, D) and string  $\alpha'$ -corrections via double field theory Josef Janyška: TBA Branislav Jurčo: TBA **Igor Khavkine**: Recursive formulas for  $L_{\infty}$  homotopy transfer to non-minimal models Martin Kolář: Catlin multitype and exotic symmetries of polynomial models in CR geometry Lukáš Krump: Multiple Dirac operators revisited Andrey Krutov: Holomorphic relative Hopf modules over the irreducible quantum flag manifolds Svatopluk Krýsl: Symplectic Twistor Operators Form Complexes Renann Lipinski Jusinskas: Asymmetrically twisted strings Jiří Nárožný: One-jets of groupoids and higher Atiyah-Lie connections Réamonn Ó Buachalla: Quantum root vectors and noncommutative Kähler structures Marián Poppr: TBA Tomáš Procházka: W algebras and integrable structures Martin Schnabl: Comments on Closed String Field Theory Jan Slovák: TBA Dana Smetanová: Mechanical systems with differential constraints and the regularity problem Filip Strakoš: Note on Geography of Bilinearized Legendrian Contact Homology of Legendrian Links Radek Suchánek: From Monge-Ampère Equations to Generalized Geometries **Josef Svoboda**: 3D *q*-series invariants and the spectrum of surface singularity David Sykes: Homogeneous 2-nondegenerate CR manifolds of hypersurface type and their modified CR symbols losef Šilhan: TBA Šimon Vedl: From shape operators of embedded manifolds to shape gauges in the theories of fundamental forces **Rikard von Unge**: A String in a Gravitational Wave Jakub Vošmera: D-branes and the exact AdS/CFT duality in three dimensions Jan Vysoký: Graded Generalized Geometry Michał Andrzej Wasilewicz: Relative tractor bundles for Lagrangean contact structures Thomas Weber: Differential Calculi on Quantum Principal Bundles over Projective Bases Henrik Winther: TBA Lenka Zalabová: TBA Fabrizio Zanello: A second order extension of the Krupka-Betounes equivalent of a Lagrangian Václav Zatloukal: Shape gauges and rotating blades — new variables for fundamental interactions? Martin Zika: NQP Manifolds, Courant Algebroids and Lagrangian Relations Vojtěch Žádník: TBA

# ABSTRACTS

#### Spyridon Afentoulidis Almpanis: Noncubic Dirac operators for finite dimensional modules

Dirac operators in Repr. Theory were used by Parthasarathy in 1972 as invariant diff. op. acting on sections of hom. vector bundles defined over G/K such that their squares give a sort of Laplacian. Parthasarathy used these operators to obtain realizations of the discrete series repr. Kostant suggested a similar Dirac op. in the case of more general spaces G/H having the same good properties as Parthasarthy's one. Kostant's operator turns out to be a particular member of a family of operators introduced by Slebarski and, if G/H is not symmetric, it is the only member of this family which squared gives the Laplacian.

In 1997, Vogan introduced an algebraic analogue of Parthasarthy's Dirac op., defined a Dirac cohomology of irreducible ( $\mathfrak{g}$ , K)-modules X using this operator and conjectured a relation between the Dirac cohomology of X and its infinitesimal character, proved by Huang and Pandzic in 2001.

In this talk, we study the algebraic analogues for the other operators of Slebarski's family.

Sandipan Bhattacherjee: Quantum Cosmology with a distinct hint of condensed matter physics The early universe is likely filled with a large number of interacting fields with unknown interactions. How can we quantitatively understand particle production (for example, during inflation and re-heating after inflation) when such fields undergo a sufficient number of non-adiabatic, non-perturbative interactions? A recent proposal of a precise mapping between stochastic particle production events during inflation and re-heating in cosmology to conduction phenomena in disordered quasi one-dimensional wires provide a powerful statistical framework to resolve such seemingly intractable calculations. In our work, we use this precise correspondence to present a derivation of the quantum corrected Fokker-Planck equation without dissipation responsible for studying the dynamical features of the stochastic particle creation events during the inflationary and re-heating stage of the universe. We also present a computation for the measure of the stochastic non-linearity arising in the inflationary and re-heating epoch of the universe, often described by the Lyapunov Exponent. Doing so, we quantify this quantum chaos arising in our set-up by a stronger measure, known as the Spectral Form Factor from the principles of Random Matrix Theory. We also discuss hints of emergent universality from Random Matrix Theory in early universe cosmology using this correspondence. Finally, we propose a bound on the Spectral Form Factor arising due to the presence of stochastic non-linear dynamical interactions in the early universe in a model-independent way.

## **Eugenia Boffo**: *BRST of the* N = 2 *superparticle and R*-*R fields*

In the worldline approach, supersymmetrizing a relativistic scalar particle can be regarded as a means to endow the particle with a spin-1/2 at the classical level (Brink-Di Vecchia-Howe 1977). Moreover, BRST quantization of a *N*-supersymmetric theory in the worldline yields the cohomology of a *N*/2-spin particle. The supersymmetric worldline approach has been used, in the N = 2 case, to find the vertices for Yang-Mills theory and compute tree level and 1-loop amplitudes (Dai-Huan-Siegel 2008). For the N = 4 superparticle, imposing a consistency condition on the BRST differential has yielded Einstein gravity and NS-NS Supergravity (Bonezzi-Meyer-Sachs 2020). On the contrary, the R-R fields, which are in a representation of the algebra in the N = 2 case, have not been studied yet. We try to fill this gap. This is on-going work with Ivo Sachs and Daniel Bockisch.

#### Ugo V. Boscain: Diffusion in almost Riemannian geometry

In this series of lectures I will define 2D-almost-Riemannian manifolds that are generalizations of 2D Riemannian manifolds in which local orthonormal frames could degenerate, but still satisfy the Hormander condition. I will discuss how to write the heat equation and the Schroedinger equation and I will study evolution through the singularities. The point of view of random walks will be of help to have an intuition of the different phenomena.

### Frederic Bourgeois: Homological invariants of Legendrian submanifolds

The aim of these lectures is to describe some of the homological invariants that are constructed in contact geometry using holomorphic curves techniques, with a focus on those for Legendrian submanifolds. Various structural properties of these invariants will be explained, as well as their applications in contact geometry.

## Andreas Čap: Tractors and mass of asymptotically hyperbolic manifolds

This talk reports on my joint work arXiv:2108.01373 with Rod Gover that is motivated by the concept of mass for asymptotically hyperbolic metrics. We work in the setting of conformally compact metrics on arbitrary manifolds with boundary (at infinity) and a class of metrics that are asymptotically hyperbolic in a weak sense and asymptotic to each other to appropriate (higher) order. This makes sure that all the metrics in the class induce the same conformal infinity on the boundary. We then associate to two metrics in the class a two-parameter family of "relative masses", that are top degree forms on the boundary with values in the standard tractor bundle of the conformal infinity.

The construction is manifestly independent of coordinates and it is easily seen to be equivariant with respect to a natural family of diffeomorphisms. Much more subtle considerations shows that a one-parameter subfamily satisfies an additional invariance conditions with respect to diffeomorphism asymptotic to the identity. Assuming that the given class contains hyperbolic metrics, this allows us to associate an invariant to a single metric in the class. In the case of hyperbolic space and the class of metrics determined by the Poincaré metric, the forms we construct can be integrated to parallel sections of the tractor bundle and this recovers that mass of asymptotically hyperbolic metrics introduced by Wang and Chrusciel-Herzlich.

## Alessandro Carotenuto: Principal pair of quantum homogeneous spaces

I will review the recently introduced notion of Principal Pairs. These objects were firstly introduced to study quantum flag manifolds and their associated quantum Poisson homogeneous spaces and they provide a simple but effective framework for producing examples of faithfully flat Hopf–Galois extensions from a nested pair of quantum homogeneous spaces. This construction is modelled on the classical situation of a homogeneous fibration  $G/N \rightarrow G/M$ , for G a group, and  $N \subseteq M \subseteq G$  subgroups. Moreover, I will present a large collection of noncommutative fibrations in the spirit of Brzeziński and Szymański

#### Goce Chadzitaskos: Some interesting properties of an asymmetric oscillator

We present a method to introduce a Fock representation for an asymmetric harmonic oscillator and the effect of "quantum beats" in the time evolution of the mean value of the position operator for the superposition of its own states. For this purpose, we study the spectrum of a quantum harmonic oscillator, which has a spring constant k— to the left of the equilibrium position and a spring of the constant k+ to the right of the equilibrium position. The symmetric harmonic oscillator is then a special case.

#### Carlo Alberto Cremonini: Cohomology of Lie Superalgebras

We study the cohomology of Lie superalgebras for the full complex of forms: superforms, pseudoforms and integral forms. We use the technique of spectral sequences to abstractly compute the Chevalley–Eilenberg cohomology and to extend some classical theorems by Koszul, as to include pseudoforms and integral forms. On one hand, these results show that the cohomology of Lie superalgebras is actually larger than expected, whereas one restricts to superforms only; on the other hand, we show the emergence of completely new cohomology classes represented by pseudoforms. These classes realise as integral form classes of sub–superstructures.

## Georgios Dimitroglou Rizell: DG-Algebraic Aspects of Contact Invariants

Contact homology is a modern contact invariant in the form of a differential graded algebra (DGA) that is generated by commuting variables corresponding to periodic Reeb orbits and non-commuting variables corresponding to Reeb chords. We review the theory about DGAs and their DG-modules, and introduce notions such as bar construction, A-infinity algebras, L-infinity algebras, and DG-homotopies, which are crucial tools when studying these invariants.

**Zdeněk Dušek**: Geodesic lemma for Finsler ( $\alpha$ ,  $\beta$ ) metrics and some special Finsler g.o. manifolds Geodesic lemma for homogeneous Finsler ( $\alpha$ ,  $\beta$ ) metrics is derived in terms of the underlying Riemannian metric  $\alpha$  and the 1-form  $\beta$ . Special Finsler g.o. manifolds which arise from a Riemannian g.o. metric  $\alpha$  are investigated and geodesic graphs are constructed.

#### Keegan Flood: The jet functor in non-commutative geometry

Consider a (possibly non-commutative) unital algebra A equipped with a first order differential calculus  $\Omega_A^1$ , i.e., a bimodule equipped with a differential satisfying the Leibniz rule. In the classical case, where  $A = C^{\infty}(M)$  for a smooth manifold M, finitely generated projective A-modules correspond to finite rank vector bundles by Serre-Swan. Hence flat (e.g., projective) left A-modules provide an analogue of vector bundles in the setting of non-commutative geometry. Our aim is to extend the notions of jet functor and jet bundle from the classical setting to non-commutative geometry. We do this by formulating the jet exact sequence, defined in terms of the differential calculus, and proving that for flat calculus and flat left A-module E the jet exact sequence for E is satisfied by a unique module  $J^k E$ . Moreover, we construct the universal prolongation operator,  $j^k : E \to J^k E$ , which allows us to extend the notion of differentiation from the calculus to the module E. This yields a notion of differential operator and we prove that various natural operations (e.g., the differential, the class of connections) are differential operators in this sense. This is based on joint work with Mauro Mantegazza and Henrik Winther.

# **Roman Golovko**: On infinite number of Hamiltonian non-isotopic exact Lagrangian fillings in high dimensions

We will discuss high-dimensional examples of Legendrian submanifolds with infinite number of exact Lagrangian fillings up to Hamiltonian isotopy.

## **Jan Gregorovič**: Construction of counterexamples to the 2-jet determination Chern-Moser Theorem in higher codimension

(Joint work F. Meylan) We first construct a counterexample of a generic quadratic submanifold of codimension 5 in  $C^9$  which admits a real analytic infinitesimal CR automorphism with homogeneous polynomial coefficients of degree 4. Then we give sufficient conditions to generate more counterexamples to the 2-jet determination Chern-Moser Theorem in higher codimension. In particular, we construct examples of generic quadratic submanifolds with jet determination of arbitrarily high order, i.e., their Tanaka prolongation takes form  $g_{-2} \oplus g_{-1} \oplus \cdots \oplus g_l$  for l of arbitrarily high order.

### Erlend Grong: Curvature in sub-Riemannian geometry

A sub-Riemannian manifold is a manifold equipped with an inner product, but this inner product is only defined on a subbundle of the tangent bundle. In other words, we can only measure length and angles in certain directions. It can be considered as a limiting case of a Riemannian manifold where the lengths of vectors outside of a certain subbundle approaches infinity.

The class of flat such manifolds have long been understood. These below to a class of nilpotent Lie groups called Carnot groups. Understanding what the analogue of a sphere or a hyperbolic space is in this geometry has proved much more complicated.

In my lecture series, I will try to give a quick introduction to sub-Riemannian manifolds and its significance to geometers, to probability theory and to control theory. From all of these areas, there has come different ideas of how to approach curvature, and we will do a quick overview in our lectures. Finally, we will focus mainly on curvature as a tool in the sub-Riemannian equivalence problem: how to determine if two sub-Riemannian manifolds differ by an isometry or not.

#### Zhangwen Guo: Weyl structures for path geometries

A geometric 2nd order ODE can be encoded as a (generalized) path geometry, which corresponds to a normal regular parabolic geometry. Choose a bundle of scales to be the line bundle defining the path geometry, each of whose nowhere-vanishing section fixes a distinguished Weyl structure. We look at the geometric information of such a Weyl structure, which includes the Weyl connection, a choice of complement to the distributions defining the geometry (which is equivalent to the soldering form), and a two-form on the geometry (which is equivalent to the Rho-tensor).

### Christoph Harrach: On boundary values of Poisson transforms for differential forms

In this talk we analyse the boundary behaviour of the image of intertwining operators for differential forms between homogeneous parabolic geometries G/P and their corresponding Riemannian symmetric spaces G/K. Explicitly, we show existence of boundary values in the cases of density valued differential forms on the conformal sphere as well as for sections of the Rumin complex on the CR-sphere, which use a conceptual method applicable to more general cases.

#### **Stanislav Hronek**: The story of O(D, D) and string $\alpha'$ -corrections via double field theory

I will review the work of me and my supervisor, Linus Wulff, over the last two years. I will briefly present the formalism of double field theory and its application to string low-energy effective action (supergravity).

To showcase the advantages of this formalism, I will prove how a special class of deformations of the supergravity background (Yang-Baxter deformations) also satisfies the supergravity equations. In particular, I will compare the difficulty of calculation with and without double field theory.

The central part of the talk will focus on string (quantum)  $\alpha'$ -corrections. I will show how one can get the first  $\alpha'$ -correction, then I will talk about the possibility of getting the second-order correction  $(\alpha')^2$  via the completion of the first-order one. Unfortunately, it seems that double field theory cannot capture all corrections since I will show that there is an obstruction at order  $(\alpha')^3$ , which means there is no  $R^4$  correction in double field theory.

#### **Igor Khavkine**: Recursive formulas for $L_{\infty}$ homotopy transfer to non-minimal models

In their well-known 1998 paper, Barnich-Fulp-Lada-Stasheff recursively lifted a Lie algebra structure to an  $L_{\infty}$  structure to a homological resolution of the underlying vector space. Their

construction relied on the acyclicity of the resolution. Using convenient notation, I generalize their recursive formulas to the non-acyclic case, that is to the case where the initial Lie algebra is replaced by a minimal  $L_{\infty}$  algebra, with non-trivial content in different cohomological degrees.

**Martin Kolář**: *Catlin multitype and exotic symmetries of polynomial models in CR geometry* We give a complete classification of symmetry algebras for polynomial models of finite Catlin multitype in  $\mathbb{C}^3$ . As a consequence, this provides complete understanding of "exotic" higher order symmetries, which violate 2-jet determination.

#### Lukáš Krump: Multiple Dirac operators revisited

I will give a survey of recent state of research in the area of multiple Dirac operators and their resolutions.

Andrey Krutov: Holomorphic relative Hopf modules over the irreducible quantum flag manifolds We construct covariant q-deformed holomorphic structures for all finitely-generated relative Hopf modules over the irreducible quantum flag manifolds endowed with their Heckenberger-Kolb calculi. In the classical limit these reduce to modules of sections of holomorphic homogeneous vector bundles over irreducible flag manifolds. For the case of simple relative Hopf modules, we show that this covariant holomorphic structure is unique.

Based on F. Díaz García, A. Krutov, R. Ó Buachalla, P. Somberg, K. R. Strung, Holomorphic relative Hopf modules over the irreducible quantum flag manifolds, Lett. Math. Phys. 111 (2021), 10, arXiv:2005.09652

#### Svatopluk Krýsl: Symplectic Twistor Operators Form Complexes

On a symplectic manifold equipped with a symplectic connection and a metaplectic structure, we define two families of sequences of differential operators, called the symplectic twistor operators. We prove that if the connection is torsion-free and Weyl-flat, the sequences in these families form complexes.

#### Renann Lipinski Jusinskas: Asymmetrically twisted strings

I will present a new class of twisted strings (based on arXiv:2108.13426), with an asymmetry between the holomorphic and antiholomorphic sectors parametrized by an integer N. Their physical content is given by the massless resonances of the closed string plus the mass-level N spectrum of the open string. The appeal of this model is the singling out of the (higher spin) massive levels of string theory together with their self/gauge/gravity interactions. Motivated by the original tree level Kawai-Lewellen-Tye relation for closed strings, its asymmetrically twisted version at four points is conjectured and shown to naturally interpolate with conventional and twisted strings. The resulting four-point amplitudes have a generalized Virasoro-Shapiro dressing factor.

#### Réamonn Ó Buachalla: Quantum root vectors and noncommutative Kähler structures

In the 2000s a series of seminal papers by Heckenberger and Kolb introduced an essentially unique covariant q-deformed de Rham complex for the irreducible quantum flag manifolds. In the years since, it has become increasingly clear that these differential graded algebras have a central role to play in the noncommutative geometry of Drinfeld–Jimbo quantum groups. Until now, however, the question of how to extend Heckenberger and Kolb's construction beyond the irreducible case has not been examined. Here we address this question for the A-series Drinfeld–Jimbo quantum groups  $U_q(\mathfrak{sl}_{n+1})$ , and show that for precisely two reduced decompositions of the longest element of the Weyl group, Lusztig's associated space of quantum root vectors gives a quantum tangent space for the full quantum flag manifold  $\mathcal{O}_q(F_{n+1})$  with associated differential graded algebra of classical dimension. Moreover, its restriction to the quantum Grassmannians recovers the q-deformed complex of Heckenberger and Kolb, giving a conceptual explanation for their origin. Time permitting, we will discuss the noncommutative Kähler geometry of these spaces and the proposed extension of the root space construction to the other series. (Joint work with P. Somberg.)

#### Tomáš Procházka: W algebras and integrable structures

I want to discuss conserved quantities and the integrable structures associated classically to one-dimensional Schroedinger operators and at the quantum level to 2d conformal field theory. There exists an infinite collection of commuting Hamiltonians, both at classical and at quantum level. Diagonalization of the quantum Hamiltonians can be done following Bazhanov, Lukyanov and Zamolodchikov by studying differential operators with two irregular singular points and a collection of apparent singularities. Virasoro algebra and its generalization Winfinity admit another set of commuting Hamiltonians related to Yangian description of the algebra. Diagonalization of this family of Hamiltonians related to Bejnamin-Ono hierarchy is trivial. Both BLZ family and BO family of Hamiltonians is connected by one-parameter family of commuting Hamiltonians associated to ILW hierarchy. These Hamiltonians can be diagonalized by Bethe ansatz equations studied by Litvinov.

## Martin Schnabl: Comments on Closed String Field Theory

I will discuss recent progress towards understanding classical solutions of Closed String Field Theory.

**Dana Smetanová**: *Mechanical systems with differential constraints and the regularity problem* The talk deals with the problem of regularity for Lagrange equations subjected to the constraints of the form of a system of ordinary differential equations of the first order. The regularity of variational systems of classical mechanics subjected to nonholonomic constraints is studied. Any constraint, which is an affine function of velocity, leads to regular equations of motion on the constraint submanifold. A similar statement does not apply to constraints quadratic at velocities. The classification of quadratic constraints in terms of the regularity of the constraint system is presented.

# **Filip Strakoš**: Note on Geography of Bilinearized Legendrian Contact Homology of Legendrian Links

The Poincaré duality for closed manifolds motivates a duality exact sequence in both linearized (due to Ekholm, Etnyre, and Sabloff) and bilinearized (due to Bourgeois, Chantraine) Legendrian contact homology setting indicating a particular asymmetry in homology. The aim of the talk is to present recent work by Bourgeois and Galant on the Geography of bilinearized Legendrian contact homology for closed Legendrians of one-jet spaces in which the duality sequence is exploited to classify the Poincaré polynomials arising from the bilinearized Legendrian contact homology. Moreover, we are to generalise those results to linked Legendrians in one-jet spaces of an arbitrary number of components.

## Radek Suchánek: From Monge-Ampère Equations to Generalized Geometries

I will present various generalized structures (such as generalized complex or generalized product structure) arising from a pair of differential forms associated with every Monge-Ampère PDE. I will show that some Monge-Ampère PDEs give rise to a pair of (anti)commuting generalized structures. As a special case, one of the structures can be a generalized (indefinite) metric, i.e. a non-degenerate generalized product structure, which then leads to metric compatible generalized geometries such as generalized Kähler or generalized chiral geometry.

#### Josef Svoboda: 3D q-series invariants and the spectrum of surface singularity

If we consider a 4-manifold W with 3-manifold M on the boundary, we can study the interplay between their numerical or q-series invariants. When W is a complex surface, certain analytic invariants can sometimes be read from the topology of M. Arnold's spectrum of singularity is too strong to be a topological invariant, nevertheless, it seems to be mysteriously related to the recent 3-manifold GPPV invariants developed using resurgence theory. I will present this relation for M being Poincaré sphere and two different generalizations of this example. Joint work in progress with L. Katzarkov, K.-S. Lee and S. Gukov.

## **David Sykes**: Homogeneous 2-nondegenerate CR manifolds of hypersurface type and their modified CR symbols

This talk is on the relationship between homogeneous 2-nondegenerate CR manifolds of hypersurface type and their modified CR symbols, a recently defined local invariant of their structure that also encodes the basic local invariants sometimes referred to as their generalized Levi forms. From this relationship we will derive algebraic criteria for a set of generalized Levi forms to admit a homogeneous model, and, in particular, obtain obstructions to homogeneity expressed in terms of the local invariants at a point. From these criteria, we obtain the classification up to local equivalence of 2-nondegenerate real hypersurfaces in  $\mathbb{C}^4$  that are locally equivalent to homogeneous CR manifolds whose symmetry groups have maximal dimension relative to their modified CR symbols. In total, there are 9 CR structures in this classification. By applying a Tanaka-theoretic algebraic prolongation to special reductions of the modified symbols we obtain estimates on the homogeneous models' symmetry group dimensions.

# **Šimon Vedl**: From shape operators of embedded manifolds to shape gauges in the theories of fundamental forces

We start by recalling the description of embedded manifolds by means of the shape operator — a bivector-valued one-form, which realizes the parallel transport by rotating (multi)vectors in the ambient Euclidean space, and which provides the curvature via a commutator product. Inspired by the embedded geometry, in a general vector bundle setup, we introduce the concept of a shape gauge. What ensues is the discussion of its existence and uniqueness. Finally, we hint at possible applications in gauge theories.

#### Jakub Vošmera: D-branes and the exact AdS/CFT duality in three dimensions

We review the recent programme initiated by Eberhardt, Gaberdiel and Gopakumar, which relates the superstring perturbation theory on  $AdS_3 \times S_3 \times \mathbb{T}^4$  with one unit of NSNS flux to the Lunin-Mathur construction of the correlators in the symmetric product orbifold of a 4-torus CFT. We extend their result to a non-perturbative regime by considering various types of D-branes in the bulk and finding their holographic duals in the boundary CFT.

#### Jan Vysoký: Graded Generalized Geometry

One can consider graded vector bundles over  $\mathbb{Z}$ -graded manifolds equipped with a non-degenerate bilinear form of an arbitrary degree. By considering a graded version of the Dorfman bracket, we arrive to a suitable set of axioms for graded Courant algebroids. We discuss graded Dirac structures and generalized complex structures and show how to use them to obtain (differential) graded Poisson and symplectic manifolds.

#### Michał Andrzej Wasilewicz: Relative tractor bundles for Lagrangean contact structures

During the talk I will discuss my recent work that provides the basis for explicitly studying relative BGG-sequences on Lagrangean contact structures. I describe the basic relative tractor

bundles associated to a Lagrangean contact structures and the relative tractor connections on them explicitly and relate them to tractor connections.

Thomas Weber: Differential Calculi on Quantum Principal Bundles over Projective Bases

In this talk we discuss a sheaf-theoretic approach to quantum principal bundles and the corresponding differential structures. For us, a quantum principal bundle is a locally cleft sheaf of comodule algebras with structure group given by a Hopf algebra and a differential calculus on the total space is a sheaf of bimodules together with a sheaf morphism, the differential, which locally satisfies the Leibniz rule. In particular, for a projective base we are able to reconstruct the space from the algebraic data, despite the algebra of global sections being trivial. We show how to induce calculi on the base space and structure Hopf algebra and, vice versa, how to build a calculus starting from differential structures on the base and structure group. As an explicit example we develop the calculus on the quantum principal bundle with total space quantum SL2, base the Riemann sphere and parabolic structure group. The results are based on a collaboration with Aschieri, Fioresi and Latini.

**Fabrizio Zanello**: A second order extension of the Krupka-Betounes equivalent of a Lagrangian We perform a geometric integration by parts of k-contact forms of degree n - s + k. As an application, we extend the Krupka-Betounes equivalent to the second order Lagrangian theories. (Joint work with M. Palese)

## **Václav Zatloukal**: Shape gauges and rotating blades — new variables for fundamental interactions?

Nature's fundamental forces can be mathematically described in terms of connections (and corresponding curvatures) on vector bundles over the spacetime. In this talk we discuss a procedure, which adds extra dimensions to a given vector bundle, thus allowing for a wider class of gauge transformations. Among the possible gauge choices we focus on what we refer to as the "shape gauge". It enjoys the property that the connection coefficients can be expressed as derivatives of a certain multivector-valued potential — the "rotating blade" — and, as a result, the curvature turns out to be given by a simple algebraic function of the shape-gauge connection. This scheme is motivated by (and analogous to) the treatment of embedded manifolds using the shape operator.

## Martin Zika: NQP Manifolds, Courant Algebroids and Lagrangian Relations

Graded manifolds naturally arise in the context of Batalin–Vilkovisky quantization as one introduces fields of non-trivial ghost degrees. Moreover, symplectic differential non-negatively graded manifolds (NQP manifolds) carry the structure given by the dynamics and gauge symmetry of certain AKSZ models. We will review the one-to-one correspondence between isomorphism classes of NQP manifolds of degree 2 and Courant algebroids known from generalized geometry following Roytenberg. Applying the construction of Lagrangian correspondences in the spirit of Weinstein's symplectic category, we can extend the one-to-one correspondence on objects to an equivalence of categories.

# GENERAL INFORMATION

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## Contact

Geometry and Physics 2022 Department of Mathematics Faculty of Sciences – Masaryk University Kotlářská 2 611 37 Brno Czech Republic

srni@math.muni.cz

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