Homological invariants of Legendrian submanifolds Lecture 2

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Geometric setup from lecture 1

Work in $Y = \mathbb{R}^{2n-1}$ with std contact structure $\xi_0 = \ker \lambda_0$,

$$\lambda_0 = dz - \sum_{i=1}^{n-1} y_i dx_i.$$

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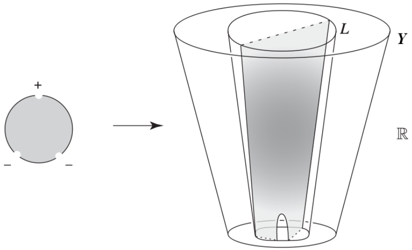
Moduli spaces of J-holomorphic disks $\mathcal{M}(c^+; c_1^-, \dots, c_\ell^-)/\mathbb{R}$.

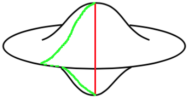


Dimension of moduli spaces

For each Reeb chord c of Λ, choose a "capping path" in Λ connecting the endpoints of c.

$\rightsquigarrow$ path of Lagrangian subspaces in $(\xi, d\lambda)$ trivial symplectic bundle.

$\rightsquigarrow$ Maslov index $\mu(c)$, if c is nondegenerate, i.e. $T_{c(0)}\Lambda \pitchfork T_{c(1)}\Lambda$ in ξ.



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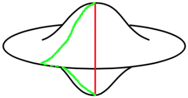

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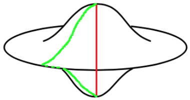

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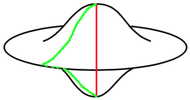

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The dimension of moduli space $\mathcal{M}(c^+; c_1^-, \dots, c_\ell^-)$ is given by

$$|c^+| - \sum_{j=1}^{\ell} |c_\ell^-|.$$

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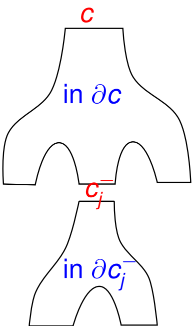

Terms in the boundary of $\mathcal{M}$ cancel in pairs.

All terms correspond to $\partial \circ \partial$.

$\rightsquigarrow \partial \circ \partial = 0$.

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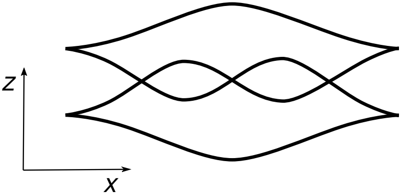
Theorem (Ekholm, Etnyre, Sullivan)

$LCH_*(\Lambda) = H_*(\mathscr{A}, \partial)$ is well-defined and invariant under Legendrian isotopy.

Example: right-handed trefoil



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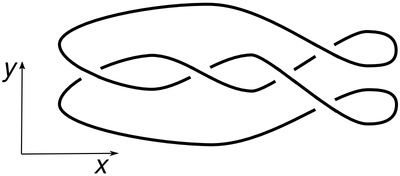

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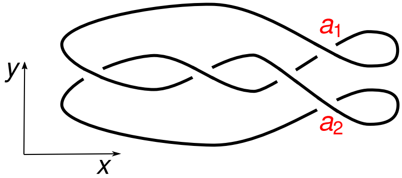

Generators a_1, a_2

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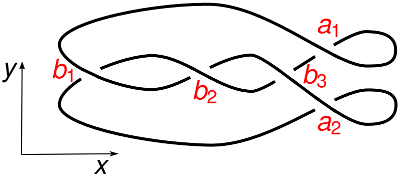

Generators a_1, a_2, b_1, b_2, b_3

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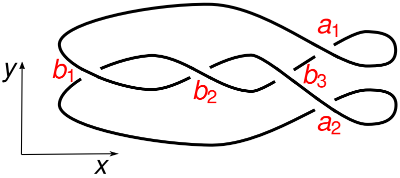

Generators a_1, a_2, b_1, b_2, b_3 with $|a_i| = 1$ and $|b_j| = 0$.

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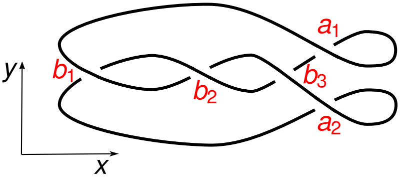

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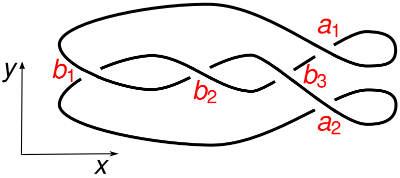

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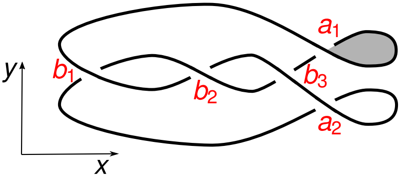

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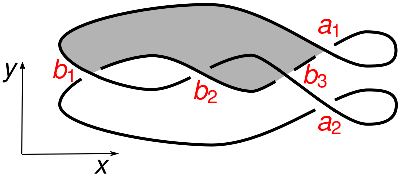

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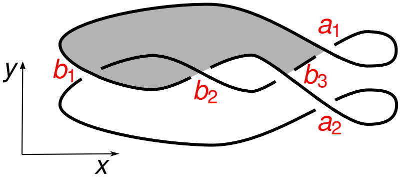

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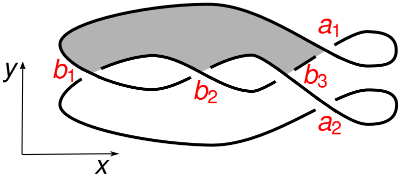

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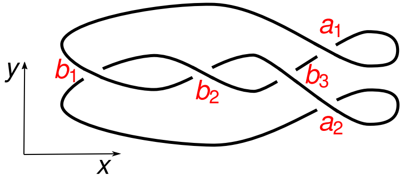

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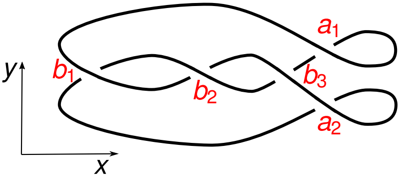

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But it is still very difficult to compute $\ker \partial$ and then $LCH(\Lambda)$!

Augmentations

Definition

An augmentation of $(\mathscr{A}, \partial)$ *is an algebra morphism* $\varepsilon : \mathscr{A} \to \mathbb{Z}_2$ *of degree* 0 *such that* $\varepsilon \circ \partial = 0$ *and* $\varepsilon(1) = 1$*.*

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If $\varepsilon_1, \varepsilon_2$ are augmentations of $(\mathscr{A}, \partial)$,
a linear map $K : \mathscr{A} \to \mathbb{Z}_2$ is an $(\varepsilon_1, \varepsilon_2)$-derivation if
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Example: Legendrian $\Lambda \subset (Y, \xi)$.
Exact lagrangian filling $L \subset (\mathbb{R} \times Y, d(e^t \lambda)) \rightsquigarrow$ augmentation ε_L.

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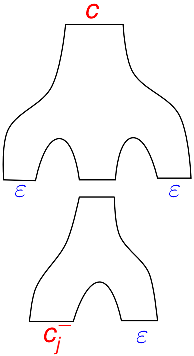
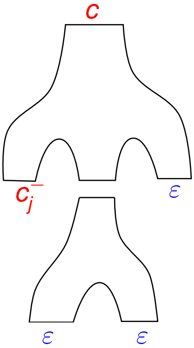


Left:
term in $\partial^\varepsilon \circ \partial^\varepsilon$.

Right:
zero term.



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$LCH_*^\varepsilon(\Lambda)$ is much easier to compute, but forgets all noncommutative content.

Example: right-handed trefoil 2

Generators a_1, a_2, b_1, b_2, b_3 with $|a_i| = 1$ and $|b_j| = 0$.

$$\begin{cases} \partial a_1 & = & 1 + b_1 + b_3 + b_3 b_2 b_1, \\ \partial a_2 & = & 1 + b_1 + b_3 + b_1 b_2 b_3. \end{cases}$$

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Augmentations:

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Are the augmentations ε_i DGA-homotopic or not?

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$\partial^\varepsilon c = \sum_{c^-} \#\mathcal{M}(c; c_1^-, \ldots, c_\ell^-)/\mathbb{R} \sum_{j=1}^{\ell} \varepsilon(c_1^-) \ldots c_j^- \ldots \varepsilon(c_\ell^-)$.

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$LCH_*^{\varepsilon_1,\varepsilon_2}(\Lambda) = H_*(C, \partial^{\varepsilon_1,\varepsilon_2})$ depends only on $[\varepsilon_1], [\varepsilon_2]$, and $\{LCH_*^{[\varepsilon_1],[\varepsilon_2]}(\Lambda) \mid \varepsilon \text{ augm. for } \lambda\}$ is invariant under Leg. isotopy.

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$LCH_*^{\varepsilon_1,\varepsilon_2}(\Lambda)$ is still convenient to compute, but also remembers some noncommutative content!

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Since $1 \neq t + 2$, all $[\varepsilon_i]$ are distinct!

Invariants of augmentations / augmentation category

Theorem (B., Galant)

Given Λ, bilinearized LCH is a complete invariant for $[\varepsilon]$.

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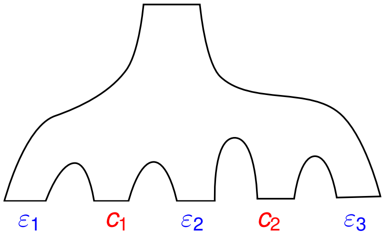

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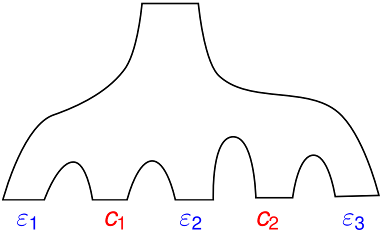

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$\rightsquigarrow$ A_∞-category, analogous to the Fukaya category in symplectic geometry.

Duality exact sequence

Theorem (Ekholm, Etnyre, Sabloff)

For any closed Legendrian $\Lambda \subset (\mathbb{R}^{2n+1}, \xi_0)$, we have a long exact sequence

$$\ldots \to H_{k+1}(\Lambda) \xrightarrow{\sigma_{n-k-1}} LCH_{\varepsilon}^{n-k-1}(\Lambda) \to LCH_k^{\varepsilon}(\Lambda) \xrightarrow{\tau_k} H_k(\Lambda) \to \ldots$$

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Original motivation: Arnold chord conjecture,
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Definition

ℓLCH-admissible polynomial: $P(t) = q(t) + p(t) + t^{n-1}p(t^{-1})$,
where q and p have non-negative integral coefficients,
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P is ℓLCH-admissible iff there exist Λ closed, connected and ε with Poincaré polynomial of $LCH^{\varepsilon}(\Lambda)$ equal to P.

Geography of (bi)linearized LCH

Definition

ℓLCH-admissible polynomial: $P(t) = q(t) + p(t) + t^{n-1}p(t^{-1})$,
where q and p have non-negative integral coefficients,
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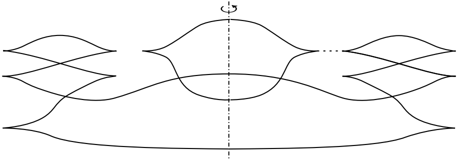

Model for $p(t) = t$.

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bLCH-admissible polynomial: $P(t) = q(t) + p(t)$, *where q and p have non-negative integral coefficients,*
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$p(-1)$ is even if n is odd and $p(-1) = 0$ if n is even.

Geography of (bi)linearized LCH

Definition

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Theorem (B., Galant)

P is bLCH-admissible iff there exist Λ closed, connected and $[\varepsilon_1] \neq [\varepsilon_2]$ with Poincaré pol. of $LCH^{\varepsilon_1,\varepsilon_2}(\Lambda)$ equal to P.