The story of $O(D, D)$ and string α'-corrections via double field theory.

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Based on works with my supervisor Linus Wulff.

Underlying theory: "Supergravity"

String theory:

$$x : \Sigma_2 \to M_{10/26}$$

Worldsheet

- Polyakov action

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}\sigma^2 \sqrt{-g}\partial_i x^m \partial_j x^n \left(g^{ij} G_{mn} + \epsilon^{ij} B_{mn}\right)$$

Target space effective action

- Background fields: (G, B, ϕ).
- Bosonic low energy string effective action:

$$S = \int \mathrm{d}x \sqrt{-G}\, e^{-2\phi} \left(R - \frac{1}{12}H^2 + \frac{1}{4}\partial^m \phi \partial_m \phi\right).$$

Supergravity equations of motion

$$\alpha' R_{mn} + 2\alpha' \nabla_m \nabla_n \phi - \frac{\alpha'}{4} H_{mkl} H_n{}^{kl} = 0 \quad (1)$$

$$-\frac{\alpha'}{2} \nabla^k H_{kmn} + \alpha' \nabla^k \phi H_{kmn} = 0 \quad (2)$$

$$-\frac{\alpha'}{2} \nabla^2 \phi + \alpha' \nabla_m \phi \nabla^m \phi - \frac{\alpha'}{24} H_{klm} H^{klm} = 0 \quad (3)$$

Double field theory (DFT) formalism

Double the coordinates $x^m \to X^M = (\tilde{x}_m, x^m)$.
$O(D,D)$ is the new structure group and the Lorentz group is doubled to $O(D-1,1) \times O(1,D-1)$
Frame-like flux formalism: $(G_{mn}, B_{mn}, \phi) \to ({E_A}^M, d)$

Definitions and formulas of the doubled fields

- Generalized vielbein:

$$ {E_A}^M = \frac{1}{\sqrt{2}} \begin{pmatrix} {e^{(+)a}}_m - e^{(+)an} B_{nm} & e^{(+)am} \\ -e^{(-)}_{am} - e^{(-)\,n}_a B_{nm} & e^{(-)\,m}_a \end{pmatrix} . $$

- Generalized dilaton:

$$ e^{-2d} = e^{-2\phi} \sqrt{-G} . $$

- Generalized fluxes:

$$ \mathcal{F}_{ABC} = 3\partial_{[A} {E_B}^M E_{C]M} \,, \qquad \mathcal{F}_A = \partial^B {E_B}^M E_{AM} + 2\partial_A d \,. $$

Action, equations and relation to ordinary supergravity

- Action:

$$S = \int \mathrm{d}X e^{-2d} \mathcal{R}$$

$$\mathcal{R} = 4\partial^a \mathcal{F}^a - 2\mathcal{F}^a \mathcal{F}^a + \mathcal{F}_a{}^{bc} \mathcal{F}_a{}^{bc} + \frac{1}{3} \mathcal{F}^{abc} \mathcal{F}^{abc}$$

- Equations of motion

$$\mathcal{R} = 0, \quad \partial^a \mathcal{F}_b + (\partial_c - \mathcal{F}_c) \mathcal{F}^a{}_{bc} - \mathcal{F}_c{}^{da} \mathcal{F}^d{}_{cb} = 0$$

- Returning to supergravity

$$\partial_M = (0, \partial_m), \quad e^{(+)} = e^{(-)} = e$$

Example of simplification using DFT

Yang-Baxter deformation for standard supergravity

- For backgrounds (G, B, ϕ) with an algebra of isometries K
- Deformation

$$\tilde{G} - \tilde{B} = (G - B)\left(1 + \eta\Theta(G - B)\right)^{-1} \tag{4}$$

$$\tilde{\phi} = \phi - \frac{1}{2} \ln \det \left(1 + \eta\Theta(G - B)\right) \tag{5}$$

Where $\Theta^{mn} = K_a^m R^{ab} K_b^n$ and R^{ab} is the R-matrix and η is the deformation parameter.

Yang-Baxter deformation in DFT

$$\tilde{E}_A{}^M = E_A{}^N \left(1 + \Theta\right)_N{}^M \tag{6}$$

$$\tilde{\mathcal{F}}_{ABC} = \mathcal{F}_{ABC}, \quad \mathcal{F}_A = \mathcal{F}_A - 2E_{AM}\nabla_N \theta^{MN} \tag{7}$$

Explored in 2007.15663.

The story of α' corrections

History

- Matsaev, Tseytlin 1987 - amplitude calculation
- Marques, Nu*ñ*ez 2015 - first correction
- Baron, Marques 2020 - higher corrections
- Hronek, Wulff 2021 - but not all of them

What are we looking for?

- Quantum α' expansion of the DFT action.
- Constructed from $\partial_A, \mathcal{F}_A, \mathcal{F}_{ABC}$
- Generalized diffeomorphism invariance
- $O(D, D)$ invariance
- Double Lorentz invariance

Corrections in standard supergravity

$$S = \int \mathrm{d}X \sqrt{-G} e^{-2\phi} \left(L_0 + \alpha' L_1 + \alpha'^2 L_2 + \alpha'^3 L_3 \right)$$

$L_1 = (\mathrm{Riem})^2 + \dots$ (bos/het)

$L_2 = (\mathrm{Riem})^3 + \dots$ (bos) $L_2 = (LCS)^2$ (het)

$L_3 = \zeta(3)\,(\mathrm{Riem})^4 + \dots$ (bos/het/type II)

Systematic approach to finding independent invariants

Double Lorentz transformations

- $\delta E_A{}^M E_{BM} = \lambda_{AB}$, parameters of the infinitesimal double Lorentz transformation
- Fluxes transform almost like connections

$$\delta \mathcal{F}_{ABC} = 3\partial_{[A}\lambda_{BC]} + 3\lambda_{[A}{}^D \mathcal{F}_{BC]D} \quad (8)$$

$$\delta \mathcal{F}_A = \partial^B \lambda_{BA} + \lambda^B{}_A F_B \quad (9)$$

- Requiring invariance $\delta L = 0$ order by order in fields, we find at leading order:
  1. $\partial^{[a}\mathcal{F}^{b]}{}_{cd} \sim \mathcal{R}^{ab}{}_{cd}$ analog of the Riemann tensor
  2. $\partial^a \mathcal{F}^a \sim \mathcal{R}$
  3. $\partial_a \mathcal{F}^b + \partial^c \mathcal{F}_a{}^{bc} \sim \mathcal{R}_a{}^b$
- Problem: the "Riemann tensor" is NOT covariant!

Results - first order α'

- Via a correction to the double Lorentz transformation

$$\tilde{\lambda}_a{}^b = \lambda_a{}^b + a\partial_a\lambda_{cd}\mathcal{F}^b{}_{cd} - b\partial^b\lambda^{cd}\mathcal{F}_a{}^{cd} .$$

- The two parameters (a, b) interpolate between the bosonic case $(a = b = -\alpha')$ and heterotic case $(a = -\alpha', b = 0)$
- We find an invariant (up to total derivatives) quantity

$$L_1 = \frac{1}{2}\mathcal{R}^{ab}{}_{cd}\mathcal{R}^{ab}{}_{cd} + \mathcal{F}^{abC}\mathcal{F}^b{}_{de}\partial_C\mathcal{F}^a{}_{de} + \Big(\partial^a\mathcal{F}^b - \mathcal{F}_c{}^{da}\mathcal{F}_c{}^{db} + \frac{1}{2}\mathcal{F}^{acd}\mathcal{F}^{bcd}\Big)\mathcal{F}^a{}_{ef}\mathcal{F}^b{}_{ef} - \frac{2}{3}\mathcal{F}^{abc}\mathcal{F}^a{}_{cd}\mathcal{F}^b{}_{de}\mathcal{F}^c{}_{ec}$$

Results - second order $(\alpha')^2$

- No independent invariant.
- Full expression found in 2009.07291 by Baron, Marquez, contains 280 terms (bosonic) and 190 terms (heterotic).
- Our work from 2109.12200: check consistency with known supergravity corrections at leading order in fields

$$L_{\text{bos}} = -\frac{1}{3}\mathcal{R}^{ab}{}_{de}\mathcal{R}^{bc}{}_{ef}\mathcal{R}^{ca}{}_{fd} + \mathcal{O}(\mathcal{F}^4), \quad L_{\text{het}} = \mathcal{O}(\mathcal{F}^4)$$

- Better approach?: $\delta'(L_1) + \delta(L_2) = 0$. Work order by order in fields and use almost covariant quantities.
- Success for the heterotic case: from 190 terms to 8 terms
- Partial success for the bosonic case: from 280 to about 25 terms.
- Should appear soon in arxiv.

Results - third order $(\alpha')^3$

- There is a known term proportional to $\zeta(3)R^4$ in standard supergravity, can we get it from DFT?
- We get agreement at leading order but an obstruction to Lorentz invariance at subleading order.
- Namely we find that it is not possible to complete R^4 with something fifth order in fields such that the sum is Lorentz invariant.
- Supervisor work in 2111.00018: use DFT to determine the B field terms up to fifth order in fields in the compactified case.

Conclusions

- DFT works very well as a simplification tool for many calculations including the first α' corrections
- However it is not able to account for all of the string α' corrections (in the non-compactified case).
- For further interest, recommended talks on youtube by Linus Wulff: "O(d,d) and α'-corrections (Linus Wulff)", "Linus Wulff, O(D,D) and string alpha'-corrections"

Thank you for the attention!

Appendix

The form of the $\mathcal{R}^4$ invariant:

$$\begin{aligned}
&R^{ab}{}_{ef}R^{bc}{}_{fg}R^{cd}{}_{he}R^{da}{}_{gh} + \tfrac{1}{2}R^{ab}{}_{ef}R^{bc}{}_{fg}R^{cd}{}_{gh}R^{da}{}_{he}\\
&-\tfrac{1}{4}R^{ab}{}_{ef}R^{ab}{}_{fg}R^{cd}{}_{gh}R^{cd}{}_{he} - \tfrac{1}{4}R^{ab}{}_{ef}R^{bc}{}_{ef}R^{cd}{}_{gh}R^{da}{}_{gh}\\
&-\tfrac{1}{8}R^{ab}{}_{ef}R^{cd}{}_{ef}R^{bc}{}_{gh}R^{da}{}_{gh} - \tfrac{1}{8}R_{abef}R^{cd}{}_{fg}R^{ab}{}_{gh}R^{cd}{}_{he}\\
&+\tfrac{1}{16}R^{ab}{}_{ef}R^{cd}{}_{ef}R^{ab}{}_{gh}R^{cd}{}_{gh} + \tfrac{1}{32}R^{ab}{}_{ef}R^{ab}{}_{ef}R^{cd}{}_{gh}R^{cd}{}_{gh}\,.
\end{aligned}$$

Appendix 2

The almost covariant quantities:

- Almost Riemmann curvature tensor

$$\mathcal{R}^{ab}{}_{cd} = 2\partial^{[a}F^{b]}{}_{cd} - F^{abe}F^{e}{}_{cd} + 2F^{[a}{}_{ce}F^{b]}{}_{ed}$$
$$\delta\mathcal{R}^{ab}{}_{cd} = 2\lambda^{[a|e|}R^{|e|b]}{}_{cd} - 2\lambda_{[c|e|}R^{ab}{}_{|e|d]} - \partial^{e}\lambda^{ab}F^{e}{}_{cd} - F_{e}{}^{ab}\partial_{e}\lambda_{cd}$$

- Almost covariant derivatives

$$D_aY^b = \partial_aY^b - F_a{}^{bc}Y^c, \qquad D^aY^b = \partial^aY^b - \frac{1}{2}F^{abc}Y^c$$
$$D_aY_b = \partial_aY_b + \frac{1}{2}F_{abc}Y_c \qquad D^aY_b = \partial^aY_b + F^{a}{}_{bc}Y_c$$

- Convenient field compositions

$$M_{ab} = F_a{}^{cd}F_b{}^{cd}, \quad M_{abc} = F_a{}^{de}F_b{}^{ef}F_c{}^{fd}, \ldots$$

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